

using the statistic

$$F = \frac{SS_{IRON}/1}{SS_E/102} = 5.99$$

From computer evaluation of the  $F$  distribution with 1 and 102 df, the  $p$ -value is less than .025. There is an effect due to the form of iron. An estimate of the difference  $\alpha_1 - \alpha_2$  is

$$\bar{Y}_{1..} - \bar{Y}_{2..} = .28$$

and a confidence interval for the difference may be obtained by noting that  $Y_{1..}$  and  $Y_{2..}$  are uncorrelated since they are averages over different observations and that

$$\text{Var}(\bar{Y}_{1..}) = \text{Var}(\bar{Y}_{2..}) = \frac{\sigma^2}{JK}$$

Thus,

$$\text{Var}(\bar{Y}_{1..} - \bar{Y}_{2..}) = \frac{2\sigma^2}{JK}$$

Estimating  $\sigma^2$  by the mean square for error,  $\text{Var}(\bar{Y}_{1..} - \bar{Y}_{2..})$  is estimated by

$$s_{\bar{Y}_{1..} - \bar{Y}_{2..}}^2 = \frac{2 \times .346}{54} = .0128$$

A confidence interval can be constructed using the  $t$  distribution with  $IJ(K - 1)$  degrees of freedom. The interval is of the form

$$(\bar{Y}_{1..} - \bar{Y}_{2..}) \pm t_{IJ(K-1)}(\alpha/2)s_{\bar{Y}_{1..} - \bar{Y}_{2..}}$$

There are 102 df; to form a 95% confidence interval we use  $t_{120}(.025) = 1.98$  from Table 4 of Appendix B as an approximation, producing the interval  $.28 \pm 1.98\sqrt{.0128}$ , or (.06, .5).

Recall that we are working on a log scale. The additive effect of .28 on the log scale corresponds to a multiplicative effect of  $e^{.28} = 1.32$  on a linear scale and the interval (.06, .50) transforms to  $(e^{.06}, e^{.50})$ , or (1.06, 1.65). Thus, we estimate that  $\text{Fe}^{2+}$  increases retention by a factor of 1.32, and the uncertainty in this factor is expressed in the confidence interval (1.06, 1.65).

The  $F$  statistic for testing the effect of dosage is significant, but this effect is expected and is not of major interest.

To test the hypothesis  $H_{AB}$  which states that there is no interaction, we consider the following  $F$  statistic:

$$F = \frac{SS_{AB}/(I-1)(J-1)}{SS_E/IJ(K-1)} = 1.17$$

From computer evaluation of the  $F$  distribution with 2 and 102 df, the  $p$ -value is .31, so there is insufficient evidence to reject this hypothesis. Thus, the deviation of the lines of Figure 12.8 from parallelism could easily be due to chance.

In conclusion, it appears that there is a difference of 6–65% in the ratio of percentage retained between the two forms of iron and that there is little evidence that this difference depends on dosage. ■