One-way Analysis of Variance

We consider a simple example that is a study of the use of a semiautomated method for measuring the amount of chlorpheniramine mealeate in tablets (Rice). We start by comparing measurements from two labs:

$$\text{Lab 1 } 4.13, 4.07, 4.04, 4.07, 4.05, 4.04, 4.02, 4.06, 4.1, 4.04$$

$$\text{Lab 2 } 3.86, 3.85, 4.08, 4.11, 4.08, 4.01, 4.02, 4.04, 3.97, 3.95$$

To study the consistency between labs of the measurement process, we could examine several statistics.

Boxplots

A visual comparison could be made with box plots. For the first lab, the quartiles are: 4.04, 4.055, and 4.07. For the second lab, the quartiles are: 3.955, 4.015, 4.07. Make side-by-side boxplots for these two labs.

Two-sample test

A two-sample test of the hypothesis that these labs have the same mean would be conducted as follows:

Lab1: mean = 4.062, sd = 0.03

Lab2: mean = 3.997, sd = 0.09

If the SDs are assumed to be different, then we would use the following test statistic:

$$\frac{4.062 - 3.997}{\sqrt{0.03^2/10 + 0.09^2/10}} = 2.17$$

The p-value for a t-distribution with 9 degrees of freedom (two-sided) is 0.06. (Based on the assumption that the observations follow a normal distribution).

If the SDs are assumed to be the same then we would compute a pooled estimate of the SD: 0.067

$$\frac{4.062 - 3.997}{0.067\sqrt{1/10 + 1/10}} = 2.17$$

The p-value for a t-distribution with 18 degrees of freedom (two-sided) is 0.04.

Multiple comparisons

We consider the case where we have 7 labs.

Notation

$Y_{i,j} = j$th measurement taken at the ith lab. Here $j = 1, \ldots, n_i$, and $i = 1, \ldots, I$.

$\bar{Y}_i =$ the mean of the n_i measurements taken at the ith lab.

Boxplots

We can still make boxplots and compare them side-by-side.

Pairs-wise comparisons

We can still compare two means at a time. There are 21 pairs to compare.

The statistic for the i_1, i_2 comparison would be:

$$\frac{\bar{Y}_{i_1} - \bar{Y}_{i_2}}{s_p\sqrt{1/10 + 1/10}}$$

where $s_p^2 = \sum_i (n_i - 1)s_i^2 / (\sum_i n_i - I)$. We would then use a t-distribution with $\sum_i n_i - I$ degrees of freedom as our test statistic.

To compensate for making 21 tests at once, our α-level would be 0.05/21.