Discrete probability review

Here are a few handy techniques for computing probabilities.

- List all of the possible outcomes in the space, which we call *Omega*. If the outcomes are equally likely then
$$P(A) \ = \ \frac{\#(A)}{\#\Omega}.$$
- Use symmetry.
- Eliminate extra details, and reduce the problem to an equivalent one.
- Consider finding the probability of the complement of the event of interest.

Let's use these techniques to answer some questions about drawing cards from a well shuffled deck of playing cards. Recall there are 52 cards in the deck, 13 red Hearts marked 2 through 10, J, Q, K, A; 13 red Diamonds; 13 black Spades; and 13 black Clubs.

Suppose we are interested in finding the probability for the following events:

$A = \{first\ card\ drawn\ is\ black\}$

$B = \{second\ card\ drawn\ is\ black\}$

$C = \{at\ least\ one\ A\ is\ drawn\}$

To find P(A) we can list all of the possible outcomes.

To find P(B) we can use symmetry:

$P(B) = P(second\ card\ is\ red) = P(B^c)$.

To find the probability of C, use the complement rule.

To do this, list the outcomes as ordered pairs (1^{st} draw, 2^{nd} draw). Then $\#\Omega = 52 * 51 = 2652$ and $\#C^c = 48 * 47 = 2256$. So $P(C)$ is $1 - 2256/2651 = .1493$.

Consider the following problems. Suppose 5 cards are drawn from the top of the deck.

- Find the chance that the ace of diamonds was drawn.

- Find the chance that the ace of diamonds and the 2 of diamonds were drawn.

- Find the chance that the first card drawn was the Ace of diamonds, the second was the 2 of diamonds, the third drawn was the 3 of diamonds, the fourth was the 4 and the fifth was the 5 of diamonds.

- Find the chance that the five cards are the 2, 3, 4, 5, and 6 of diamonds (in any order).

- Find the chance that 3 of the cards are red.

Simple Random Sampling

Now let's transfer what you have know about drawing card from a deck to simple random sampling.

Deck of cards	Stat 135 class
Cards are marked A-K	Students are ordered 1-N (70 in our case)
Shuffle the deck	Randomize the order of the students
Deal a card from the deck	Pick a student from the class
Deal n cards from the deck	Select a sample of n students
The chance the Ace of diamonds is picked	The chance the first student chosen is #1.
The number of possible "hands" dealt	The number of distinct samples drawn.

Consider the following problems. Suppose 5 students are chosen at random from the class.

- Find the chance that student #37 was chosen.

– Find the chance that #37 and #38 were in the sample.

– Find the chance that the first student selected for the sample was #1, the second was #2, . . . , and the the fifth was student #5.

– Find the chance that the five students in the sample are #1, 2, 3, 4, 5.

– Find the chance that 3 of the students have spans larger than 20 cm. – Hang on first we need the following information:

Here are the right-hand spans (in cm) of the students in our class.

Span (cm)	Count
17	1
18	8
18.5	3
19	8
19.5	4
20	9
20.5	2
21	11
21.5	2
22	6
22.5	4
23	6
23.5	1
24	2
24.5	1
25	2

Consider our class as a population. Make a histogram of the population values for hand-span.