

METHOD OF MAXIMUM LIKELIHOOD

Another procedure for estimating parameters, it views the pdf or pmf as a function in the parameter given the observed values. This function is called the likelihood, and if is maximized with respect to the parameter to provide an estimate of the parameter.

Example X has a binomial(100, p) distribution, and it a value of 40 is observed for X .

Find the likelihood function – a function of p

$$L(p) = P(X = 40|p) =$$

Maximize it with respect to p .

$$\partial L(p)/\partial p =$$

Establish that the value found is indeed a maximum of the likelihood.

Often it is easier to maximize the log of the likelihood function, than to maximize the likelihood function itself. consider the normal example, where Y_1, \dots, Y_n are iid normal with a mean μ and a variance σ^2 . Suppose we observe values, x_1, \dots, x_n and we wish to estimate μ .

In this example, we have a continuous pdf rather than a pmf.

Find the likelihood function – a function of μ

$$P(X_1 \in dx_1, \dots, X_n \in dx_n | \mu) = f(x_1, \dots, x_n | \mu) dx_1 \cdots dx_n =$$

We ignore the infinitesimal region and maximize the likelihood function:

$$L(\mu) = f(x_1, \dots, x_n | \mu)$$

Alternatively we could maximize the log-likelihood function (WHY?):

$$l(\mu) = \log(f(x_1, \dots, x_n | \mu))$$

Maximize it with respect to μ .

$$\partial l(\mu) / \partial \mu =$$

Establish that the value found is indeed a maximum of the likelihood.