Monotone Games
Network Architecture, Salience and Coordination

To be presented at Social Choice and Networks
CS 294 / Econ 207A / Math C223A / Stat 206A
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Introduction

The prisoner's dilemma game with one-shot payoffs

	C	D
C	$2, 2$	$0, 3$
D	$3, 0$	$1, 1$

has a unique Nash equilibrium in which each player chooses D (defection), but both player are better if they choose C (cooperation).

If the game is played repeatedly, then (C, C) accrues in every period if each player believes that choosing D will end cooperation (D, D), and subsequent losses outweigh the immediate gain.

- The Folk Theorem for infinitely repeated games demonstrates that cooperation can be sustained in long run relationships.

- But the Folk Theorem is only partly successful as a theory of cooperative behavior.

- It guarantees the existence of a large class of equilibria, some of which are efficient and many more of which have unattractive welfare properties.

- One response is to introduce more structure to guarantee efficient equilibrium outcomes in repeated games.

- A monotone game is an extensive-form game with simultaneous moves and an irreversibility structure on strategies.

- It captures a variety of situations in which players make partial commitments.

- We characterize conditions under which equilibria result in efficient outcomes.

- The game has many equilibrium outcomes so the theory lacks predictive power.

- To produce stronger predictions, we restrict attention to sequential equilibria, or Markov equilibria, or symmetric equilibria.

- Whether any of these refinements is reasonable in practice is an empirical question.

- Multiple equilibria cannot be avoided in general and the theory cannot tell us which equilibrium is most likely to be played.

- Identify the important factors in creating the “salience” of certain equilibria.

The game

- An indivisible public project with cost K and N players, each of whom has an endowment of E tokens.

- The players simultaneously make *irreversible* contributions to the project at a sequence of dates $t = 1, ..., T$.

- The project is carried out if and only if the sum of the contributions is large enough to meet its cost.

- If the project is completed, each player receives A tokens *plus* to the number of tokens retained from his endowment.

The game is defined by five parameters (positive integers except for $A \geq 0$)

- A - value the public good
- E - initial individual endowment
- K - cost of the public good
- N - number of players
- T - number of periods.

Each of these parameters influences the set of equilibria of the game in a distinct way.

To avoid trivialities, we assume that

- the aggregate endowment is greater than the cost of the project (completion is feasible)

$$NE > K$$

- the aggregate value of the project is greater than the cost (completion is efficient)

$$NA > K$$

- the project is not completed by a single player (either it is not feasible or it is not rational)

$$\min\{A, E\} < K.$$

Information structure

- To complete the description of the game, we have to specify the information available to each player.

- Perfect information makes it easier for players to coordinate their actions, if they are so inclined.

- In the absence of perfect information, players beliefs play a larger role in supporting (possibly inefficient) equilibria.

- Asymmetry of the information structure may have an impact on the "selection" of equilibria.

- The information structure is represented by a *directed* graph (or network).

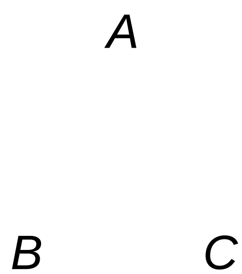
- Each player is located at a node of the graph and player i can observe player j if there is an edge leading from node i to node j.

- The experiments involve three-person networks: empty, complete and all networks with one or two edges.

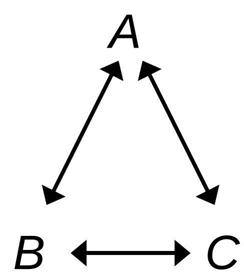
- Each network has a different architecture, a different set of equilibria, and different implications for the play of the game.

Networks

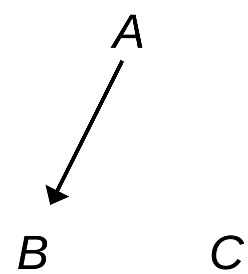
<u>Empty</u>



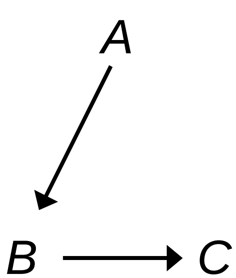
<u>Complete</u>



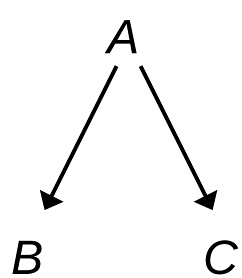
<u>One-link</u>



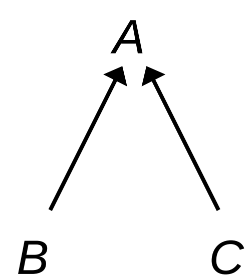
<u>Line</u>



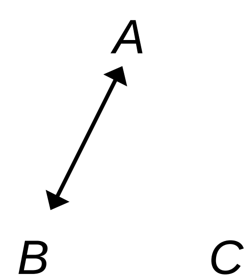
<u>Star-out</u>



<u>Star-in</u>



<u>Pair</u>



The empty network

The game is essentially the same as the static game in which all players make simultaneous binding decisions.

Proposition (one-shot) *(i) There exists a pure-strategy Nash equilibrium with no completion. Conversely, there exists at least one pure-strategy equilibrium in which the project is completed with probability one. (ii) The game also possesses a symmetric mixed-strategy equilibrium in which the project is completed with positive probability.*

The indivisibility of the public project makes each contributing player "pivotal" (Bagnoli and Lipman (1992)).

The complete network

The sharpest result is obtained for the case of pure-strategy sequential equilibria.

Proposition (pure strategy) *Suppose that* $A > E$ *and* $T \geq K$. *Then, under the maintained assumptions, in any pure strategy sequential equilibrium of the game, the public project is completed with probability one.*

In any pure strategy equilibrium, the probability of completion is either zero or one, so it is enough to show that the no-completion equilibrium is not sequential.

The logic of the proof can be illustrated by an example

$$N = 3, A > 1, E = 1, K = T = 2.$$

Suppose, contrary to the claim, that there exists a pure sequential equilibrium with zero provision so every player's payoff is simply the value of his endowment $E = 1$.

If one player contributes his token at date 1, one of the remaining players can earn at least $A > 1$ by contributing his endowment at date 2.

Thus, the good must be provided at date 2 if one player contributes at date 1. Anticipating this response, it is clearly optimal for someone to contribute a unit at date 1.

Mixed strategies expand the set of parameters for which there exists a no-completion equilibrium.

Proposition (mixed strategy) *Suppose that* $A > E$ *and* $T \geq K$. *Then there exists a number* $A^*(E, K, N, T)$ *such that, for any* $E < A < A^*$ *there exists a mixed strategy equilibrium in which the project is completed with probability zero.*

The use of mixed strategies in the continuation game can discourage an initial contribution and support an equilibrium with no completion.

The example will make this clear:

As long as $A > 1$ there is no pure-strategy sequential equilibrium in which the good is not provided.

With mixed strategies, if one player contributes a token in the first period, the continuation game possesses a symmetric mixed-strategy equilibrium.

A necessary and sufficient condition for $0 < \lambda < 1$ to be an equilibrium strategy is that each player be indifferent between contributing and not contributing $A = \lambda A + 1$.

In this mixed-strategy equilibrium, the good is provided unless neither of the two players contributes, that is, the good is provided with probability $1 - (1 - \lambda)^2$.

If the player who contributes at the first period anticipates his opponents will play the symmetric mixed-strategy equilibrium at the second period, it is rational for him to contribute if

$$\left[1 - (1 - \lambda)^2\right] A \geq 1$$

or

$$A^2 - A - 1 \geq 0.$$

Symmetric Markov-perfect equilibria (SMPE)

The class of SMPE takes a relatively simple form. The main predictions from SMPE can be summarized by four facts:

- There are no pure strategy SMPE, although mixed strategies may only be used off the equilibrium path.

- There is no completion of the public project in early periods when A "high" and no provision at all when A "low."

- The contribution probability at each state when A is "high" is at least as high as when A is "low."

- A game with horizon $T < T'$ is isomorphic to a continuation game starting in period $T' - T$ of the game with horizon T'.

A binary game ($E = 1$) example

$$A = 3, E = 1, K = 2, N = 3, T = 5$$

τ / n	0	1
4	0.00	$--$
3	0.00	0.00
2	0.00	0.00
1	0.56| 0.55| 0.00	0.00
0	0.00| 0.21| 0.79	0.67

where n is the total number of contributions and τ is the number of periods remaining after the current period.

$$A = 1.5, E = 1, K = 2, N = 3, T = 5$$

τ/n	0	1
4	0.00	$--$
3	0.00	0.00
2	0.00	0.00
1	0.00	0.00
0	0.00	0.33

The Markov property reduces the set of sequential equilibria, sometimes substantially.

Frequencies of contribution in the complete network

$A=3, E=1, K=2, N=3$

τ / n	0	1	2
4	0.09 (270)		
3	0.08 (207)	0.11 (38)	0 (2)
2	0.11 (165)	0.07 (54)	0.25 (8)
1	0.37 (117)	0.07 (76)	0.10 (10)
0	0.36 (36)	0.60 (94)	0.08 (24)

τ / n	0	1	2
1	0.18 (270)		
0	0.62 (159)	0.54 (54)	0 (9)

( ) - # of obs.

$A = 1.5,\ E = 1,\ K = 2,\ N = 3$

τ / n	0	1	2
4	0.09 (270)		
3	0.05 (207)	0.03 (36)	0 (3)
2	0.06 (177)	0.06 (54)	0.25 (4)
1	0.26 (144)	0.19 (70)	0.17 (6)
0	0.20 (57)	0.48 (88)	0.09 (23)

τ / n	0	1	2
1	0.18 (270)		
0	0.35 (150)	0.33 (64)	0 (7)

( ) - # of obs.

Markov behavior

An interesting question is whether subjects' behavior is consistent with Markov strategies.

- Consider states $(1, 2), (1, 1), (1, 0)$ where one token has already been contributed. At each state (n, τ) there are $(4 - \tau)$ distinct histories reaching the state (n, τ).

- These histories are denoted by $h(t)$, where $h(t)$ represents the history where one token was contributed at time period t.

- The (joint) null hypothesis at each of the three states is that the relative frequencies of contributions from different histories reaching the current state are equivalent.

The relative frequencies of contributions from the different histories

$E=1,\ K=2,\ N=3,\ T=5$

A	(n,τ)	$h(1)$	$h(2)$	$h(3)$	$h(4)$	p-value
1.5	(1,2)	0.03 (34)	0.10 (20)	–	–	0.63
	(1,1)	0.06 (32)	0.25 (16)	0.32 (22)	–	0.05
	(1,0)	0.54 (28)	0.25 (8)	0.30 (10)	0.52 (42)	0.30
3	(1,2)	0.00 (30)	0.17 (24)	–	–	0.07
	(1,1)	0.00 (30)	0.06 (18)	0.14 (28)	–	0.21
	(1,0)	0.47 (30)	0.75 (18)	0.60 (20)	0.64 (28)	0.27

$E=2,\ K=2,\ N=3,\ T=5$

A	(n,τ)	$h(1)$	$h(2)$	$h(3)$	$h(4)$	p-value
1.5	(1,2)	0.56 (18)	0.45 (22)	–	–	0.25
	(1,1)	0.00 (10)	0.05 (20)	0.10 (40)	–	0.50
	(1,0)	0.50 (10)	0.33 (18)	0.47 (32)	0.31 (32)	0.12
3	(1,2)	0.05 (44)	0.00 (6)	–	–	0.10
	(1,1)	0.11 (38)	0.00 (6)	0.06 (16)	–	0.60
	(1,0)	0.43 (30)	0.67 (6)	0.57 (14)	0.41 (34)	0.53

Equilibrium behavior

- The more refinements are satisfied, the fewer equilibria we have to consider.

- If we add symmetry to the Markov property, we are led to consider the ability of the SMPE to account for the data.

- Symmetry is inconsistent with pure-strategy equilibria with positive provision (two players contribute and one does not).

- A SMPE is necessarily a mixed-strategy equilibrium if there is positive provision.

Quantal Response Equilibrium (QRE)

- Players choose responses with higher expected payoffs with higher probability – better response instead of best responses.

- Players have rational expectations and use the true mean error rate when interpreting others' actions.

- Provide a statistical framework (structural econometric approach) to analyze game theoretic data (field and laboratory).

- If Nash had been a statistician, he might have discovered QRE rather then Nash equilibrium – Colin Camerer –

In practice, QRE often uses a logit or exponentiation payoff response function:

$$\Pr(a_i) = \frac{\exp[\lambda \sum_{a_{-i} \in A_{-i}} \Pr(a_{-i}) u_i(a_i, a_{-i})]}{\sum_{a'_i \in A_i} \exp[\lambda \sum_{a_{-i} \in A_{-i}} \Pr(a_{-i}) u_i(a'_i, a_{-i})]}.$$

The choice of action becomes purely random as $\lambda \to 0$, whereas the action with the higher expected payoff is chosen for sure as $\lambda \to \infty$.

QRE estimation results and the probability of contribution

A =3, *E* =1, *K* =2, *N* =3

β=10.05 (0.78), Log_lik =-472.52

τ / n	0	1	2
4	0.11		
3	0.14	0.07	0.00
2	0.18	0.10	0.00
1	0.20	0.17	0.00
0	0.75	0.65	0.00

β = 10.51 (1.27), Log_lik = -278.55

τ / n	0	1	2
1	0.19		
0	0.76	0.65	0

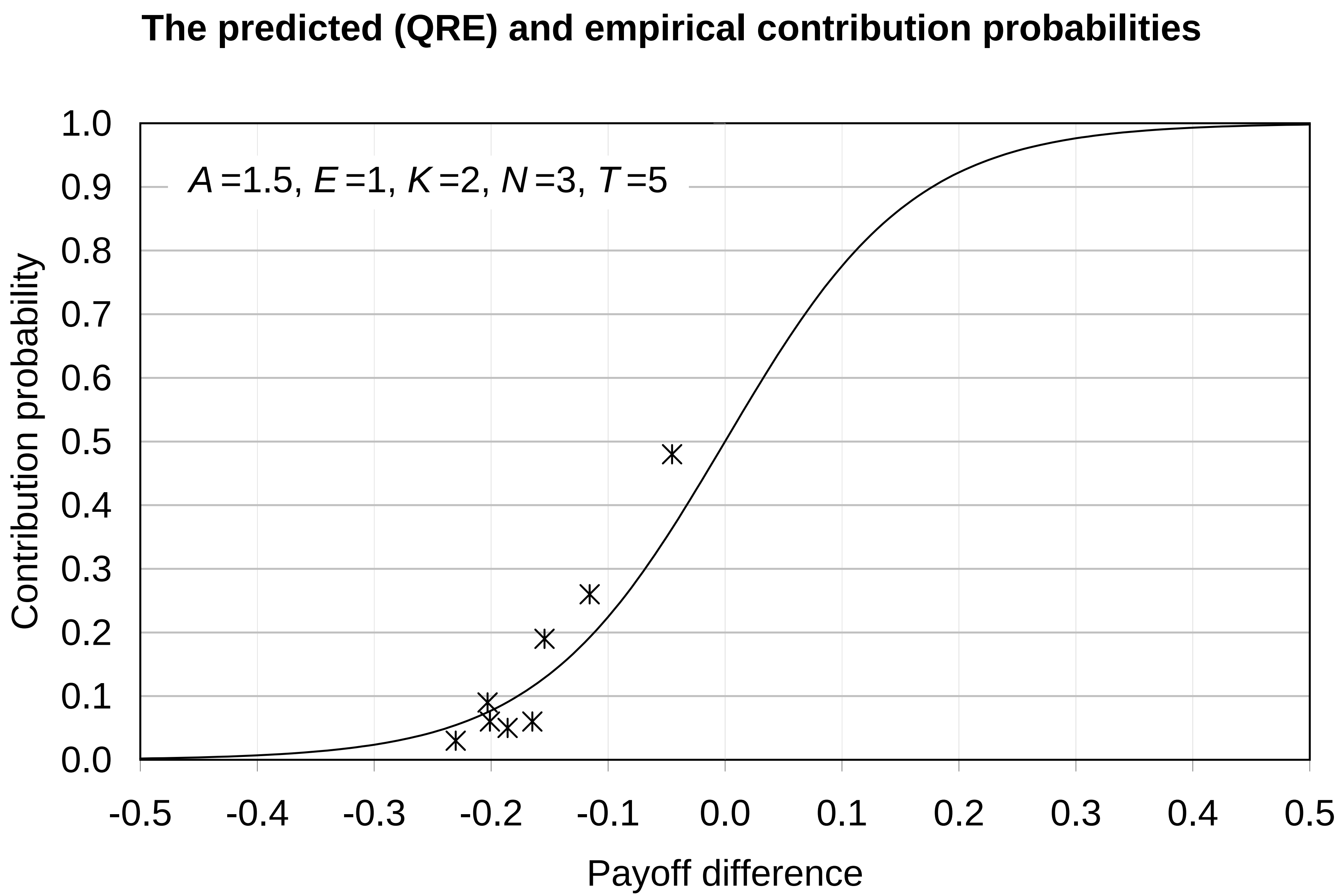
$A=1.5$, $E=1$, $K=2$, $N=3$

β=12.34 (0.83), Log_lik =-475.01

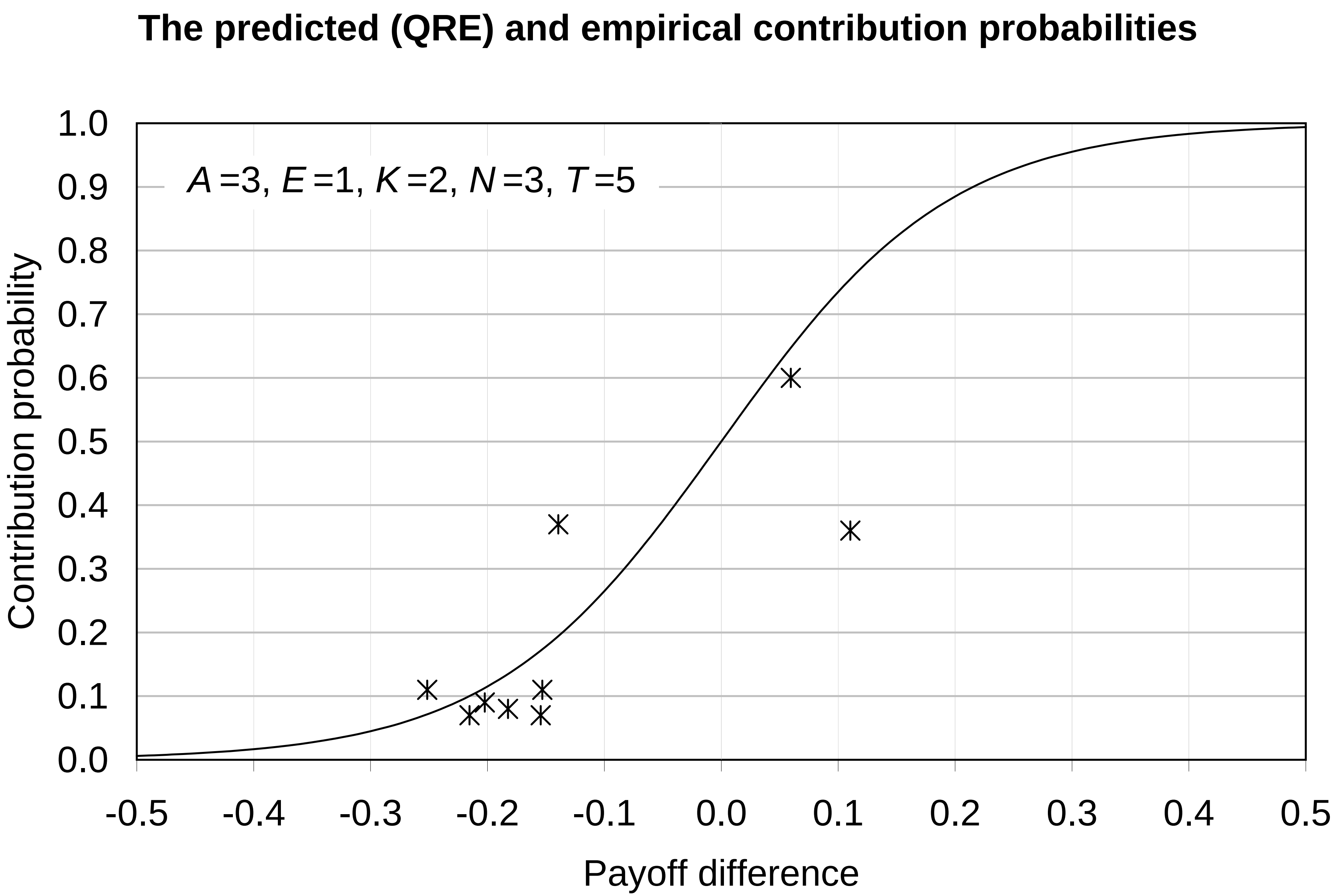
τ / n	0	1	2
4	0.08		
3	0.09	0.06	0.00
2	0.12	0.08	0.00
1	0.19	0.13	0.00
0	0.00	0.36	0.00

β = 2.26 (0.20), Log_lik = -296.41

τ / n	0	1	2
1	0.4		
0	0.3	0.42	0.09



The predicted (QRE) and empirical contribution probabilities
A =1.5, E =1, K =2, N =3, T =5
Contribution probability
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.5
-0.4
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
0.4
0.5
Payoff difference


The predicted (QRE) and empirical contribution probabilities
A =3, E =1, K =2, N =3, T =5
Contribution probability
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.5
-0.4
-0.3
-0.2
-0.1
0.0
0.1
0.2
0.3
0.4
0.5
Payoff difference

The one-link network

Adding one link to the empty network creates a simple asymmetry among the three players.

Proposition (one link) *Suppose that* $A > E = 1$ *and* $T \geq K = 2$*. Then, under the maintained assumptions, every pure-strategy sequential equilibrium completes the public project with probability one.*

Equilibria in which B contributes first and A contributes after observing B contribute seem "salient."

The line, star-out, star-in and pair networks

The remaining networks can each be obtained by adding a single link to the one-link network.

Proposition (networks) *Suppose that* $A > E = 1$ *and* $T \geq K = 2$. *Then, sequential rationality implies completion of the public project (with positive probability) in all of the networks except the empty network.*

Our focus in the sequel is to identify the impact of network architecture on efficiency and dynamics.

Conjectures / hypotheses

- Uninformed players make a contribution early in the game to encourage other players to contribute.

- Informed players delay their contributions until they have observed another player contribute.

- Players who are symmetrically situated in a network have difficulty coordinating on an efficient outcome.

- Players who are otherwise similarly situated behave differently in different networks.

The frequencies of contributions in the one-link network

$A=2,\ E=1,\ K=2,\ N=3,\ T=3$

		A		*B*	*C*
1	n_i	--		--	--
	Freq.	0.104 (135)		0.570 (135)	0.163 (135)
2	n_i	0	1	--	--
	Freq.	0.039 (51)	0.500 (70)	0.345 (58)	0.035 (113)
3	n_i	0	1	--	--
	Freq.	0.222 (36)	0.583 (48)	0.158 (38)	0.046 (109)

( ) - # of obs.

The frequencies of contributions in the line network

A =2, *E* =1, *K* =2, *N* =3, *T* =3

		A		*B*		*C*
1	n_i	--		--		--
	Freq.	0.006 (180)		0.172 (180)		0.900 (180)
2	n_i	0	1	0	1	--
	Freq.	0.007 (148)	0.161 (31)	0.077 (13)	0.632 (136)	0.167 (18)
3	n_i	0	1	0	1	--
	Freq.	0.115 (61)	0.045 (112)	0.182 (11)	0.686 (51)	0.200 (15)

( ) - # of obs.

The frequencies of contributions in the star-out network

$A=2$, $E=1$, $K=2$, $N=3$, $T=3$

		A			*B,C*
1	n_i	--			--
	Freq.	0.006 (165)			0.318 (330)
2	n_i	0	1	2	--
	Freq.	0.027 (73)	0.195 (77)	0.071 (14)	0.187 (225)
3	n_i	0	1	2	--
	Freq.	0.089 (45)	0.922 (77)	0.000 (24)	0.044 (183)

( ) - # of obs.

The frequencies of contributions in the star-in network

A=2, E=1, K=2, N=3, T=3

		A	B,C	
1	n_i	--	--	
	Freq.	0.620 (150)	0.157 (300)	
2	n_i	--	0	1
	Freq.	0.439 (57)	0.080 (100)	0.229 (153)
3	n_i	--	0	1
	Freq.	0.094 (32)	0.173 (52)	0.215 (158)

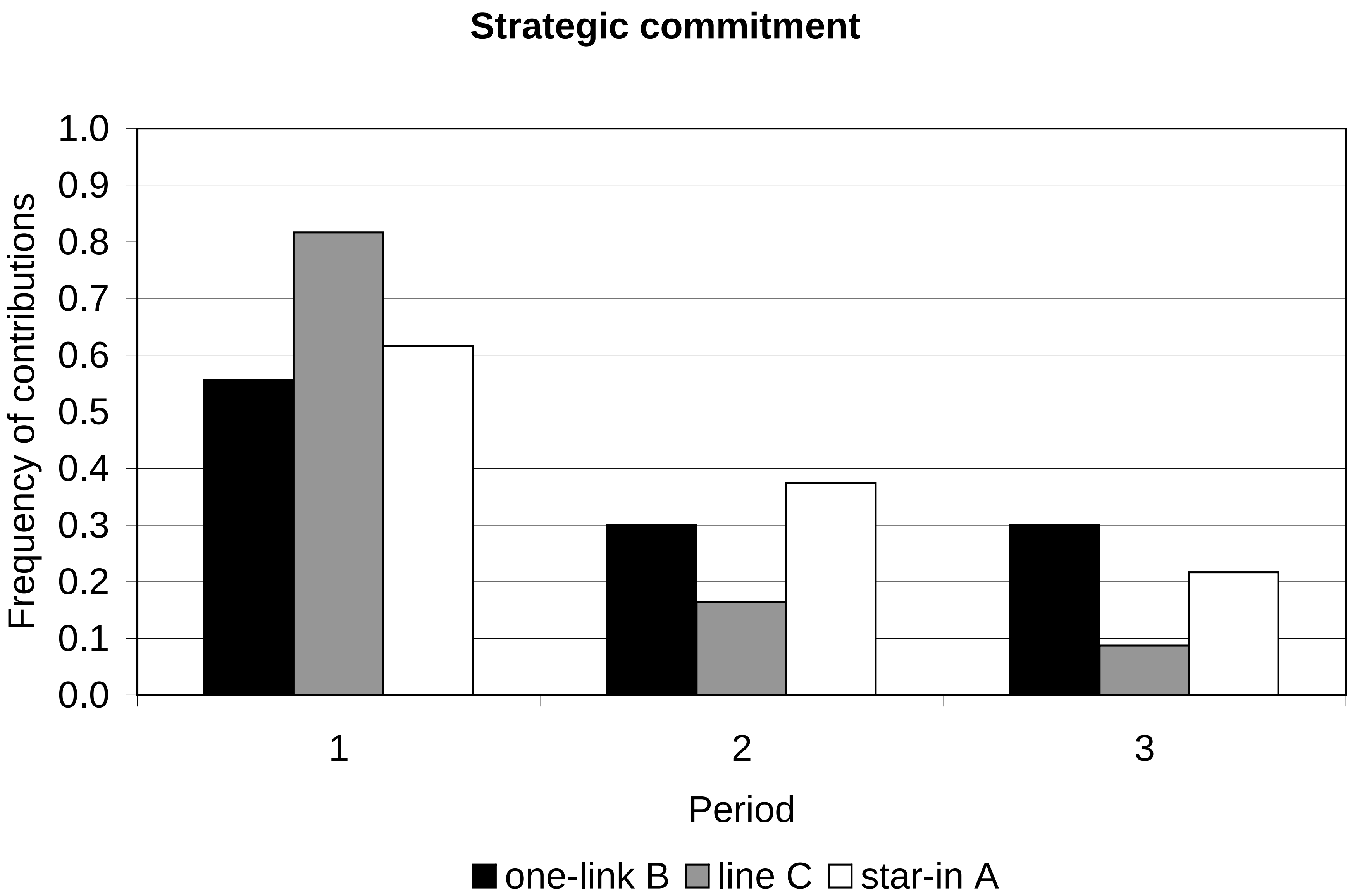
( ) - # of obs.

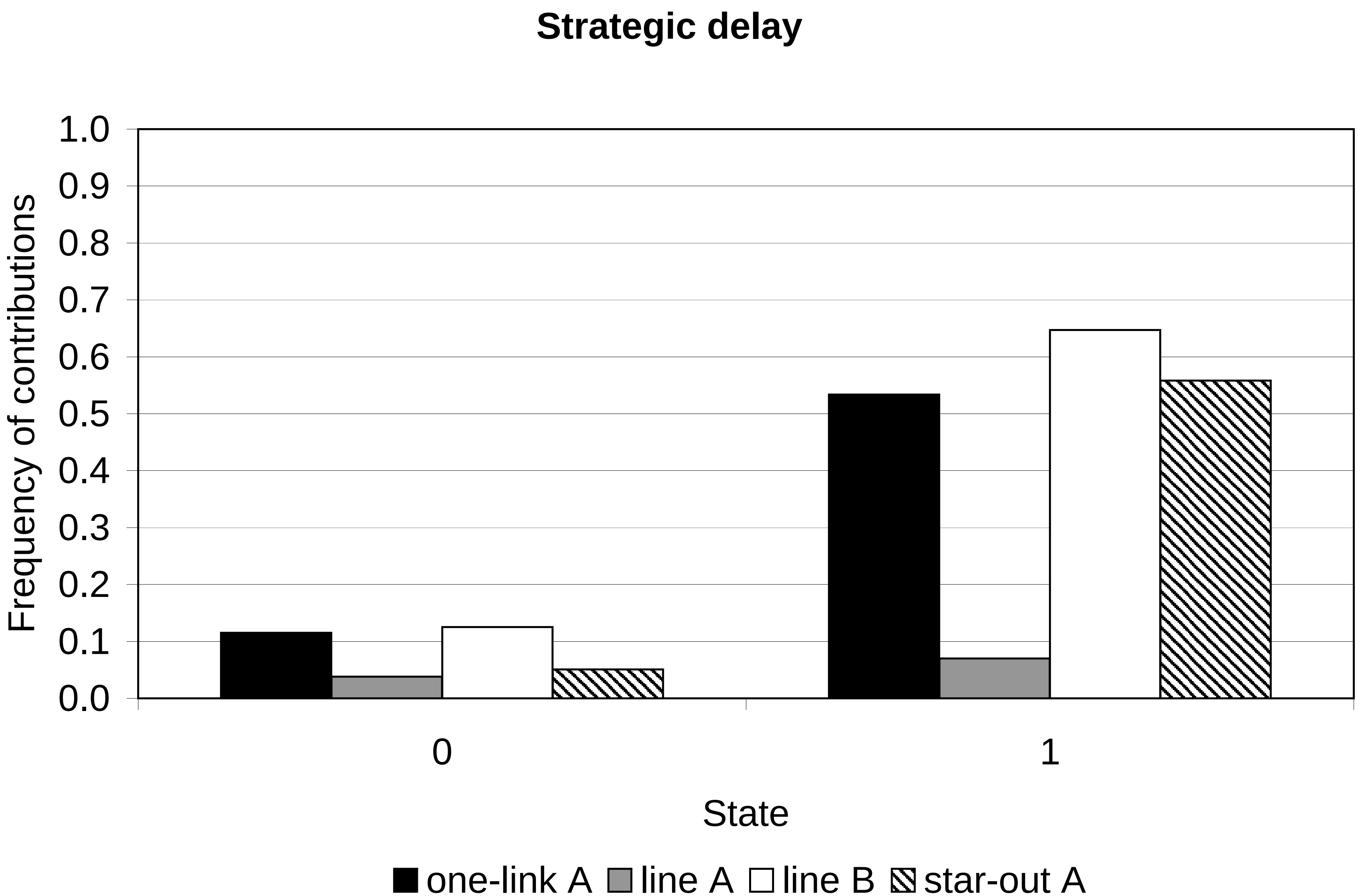
The frequencies of contributions in the pair network

$A=2$, $E=1$, $K=2$, $N=3$, $T=3$

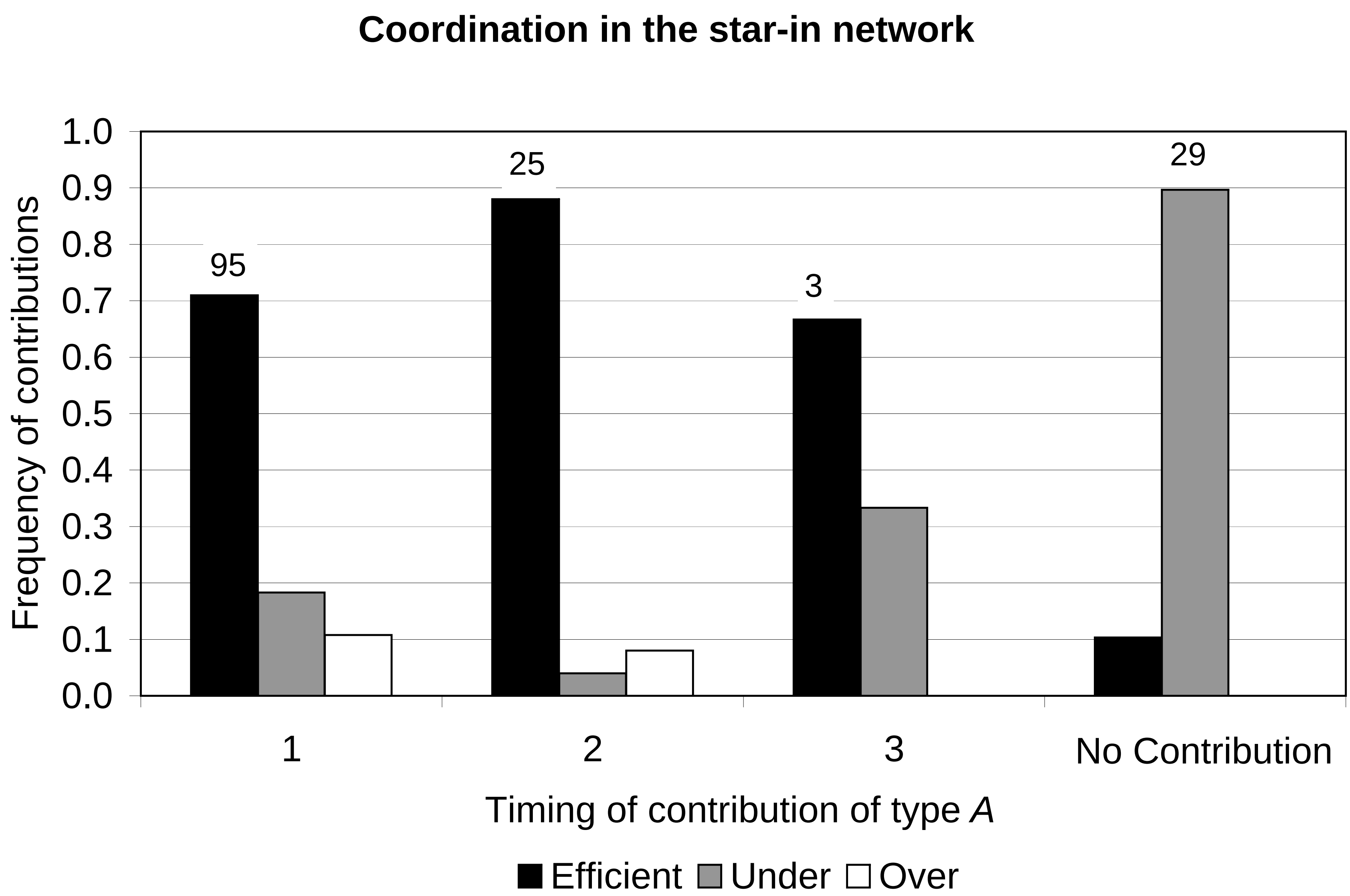
		A, B		C
1	n_i	--		--
	Freq.	0.300 (300)		0.100 (150)
2	n_i	0	1	--
	Freq.	0.327 (156)	0.426 (54)	0.022 (135)
3	n_i	0	1	--
	Freq.	0.333 (105)	0.419 (31)	0.053 (132)

( ) - # of obs.


Strategic commitment
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Frequency of contributions
1
2
3
Period
one-link B
line C
star-in A



Strategic delay
Frequency of contributions
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
State
one-link A
line A
line B
star-out A



Coordination in the star-in network
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Frequency of contributions
95
25
3
29
1
2
3
No Contribution
Timing of contribution of type A
Efficient
Under
Over

Conclusions

- The architecture induces the use of strategic delay by some players and the use of strategic commitment by others.

- These in turn facilitate certain behaviors – and possibly certain equilibria – salient.

- Asymmetry gives rise to salience which, in turn, is an aid to predictability and coordination.

- These regularities lack a proper theoretical explanation – puzzles for game theorists to ponder.

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2. Gale D. (2001) “Monotone Games with Positive Spillovers.” *Games and Economic Behavior* **37**, pp. 295-320.

3. Choi S., D. Gale and S. Kariv (2008) “Sequential Equilibrium in Monotone Games: Theory-Based Analysis of Experimental Data.” *Journal of Economic Theory* **143**, pp. 302–330.

4. Choi S., D. Gale, S. Kariv and T. Palfrey (2010) “Network Architecture, Salience and Coordination.”