Probability Seminar

Organizer: Tai Melcher & George Kordzakhia

Wednesday, 3:10–4:00pm, 330 Evans

Jan 25 **Jonathan Taylor**, Stanford

Critical values of smooth random fields and eigenvalues of random matrices

We discuss the generic behaviour of the critical points / values of smooth Gaussian random fields on smooth manifolds $f : M \to \mathbb{R}$, which we think of as point processes on the parameter space of the field (the critical points) with real-valued marks (the critical values).

For parameter spaces of a fixed dimension, these marked point processes can be combined with some tools from differential topology to derive an accurate approximation to the supremum distribution

$$\mathbb{P}\left\{\sup_{x \in M} f(x) \geq u\right\}$$

based on the geometry of the excursion sets

$$\{x \in M : f(x) \geq u\},$$

specifically the expected Euler characteristic of the excursion sets.

In general, the accuracy of the above expression is poorly understood if we allow the dimension of M to grow. In this work, we investigate some aspects of the accuracy in the high dimensional setting, restricting attention to isotropic process on $[0,1]^n$ with n growing. In this situation, the "spectrum" of critical values behave in some sense like the eigenvalues of a large GOE (Gaussian Orthogonal Ensemble) matrix at the bulk and the edge.

We identify two separate regimes for the behaviour of the mean spectral measure of the critical values of smooth isotropic Gaussian fields in high dimensions. Understanding the limiting behaviour of the mean spectral measure depends on a characterization of the covariance functions of isotropic processes in high dimensions.