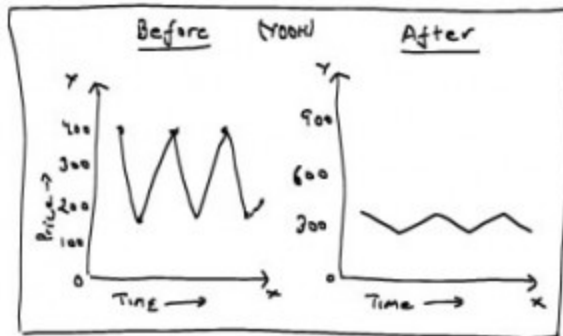


Detecting Seasonal Volatility: A Banking Application

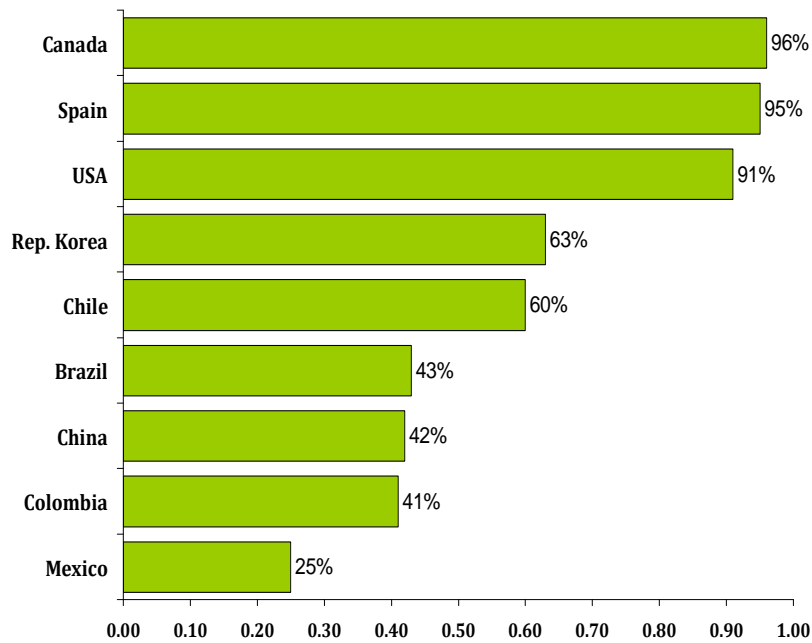


As you can see, we were finally able to control the volatility in our stock price. This was done by carefully increasing the Y-axis intervals.

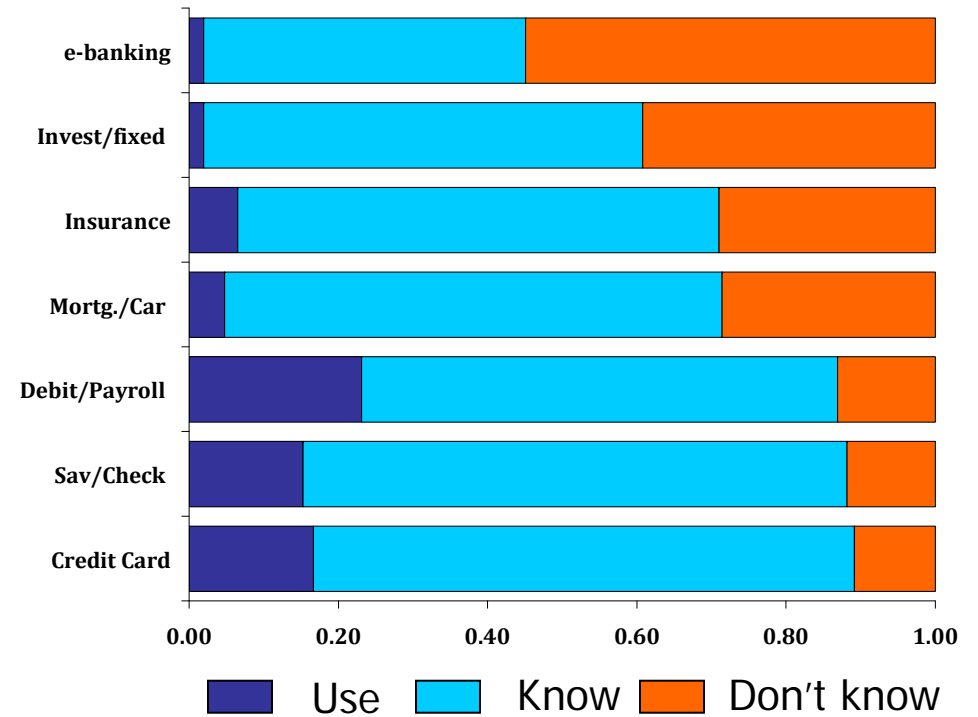
Irma Hernandez Magallanes
University of California, Berkeley
Banco de México

Use of cash in Mexico

Percentage of users of Financial Services¹



Given that Mexicans know the services, have they used it?²



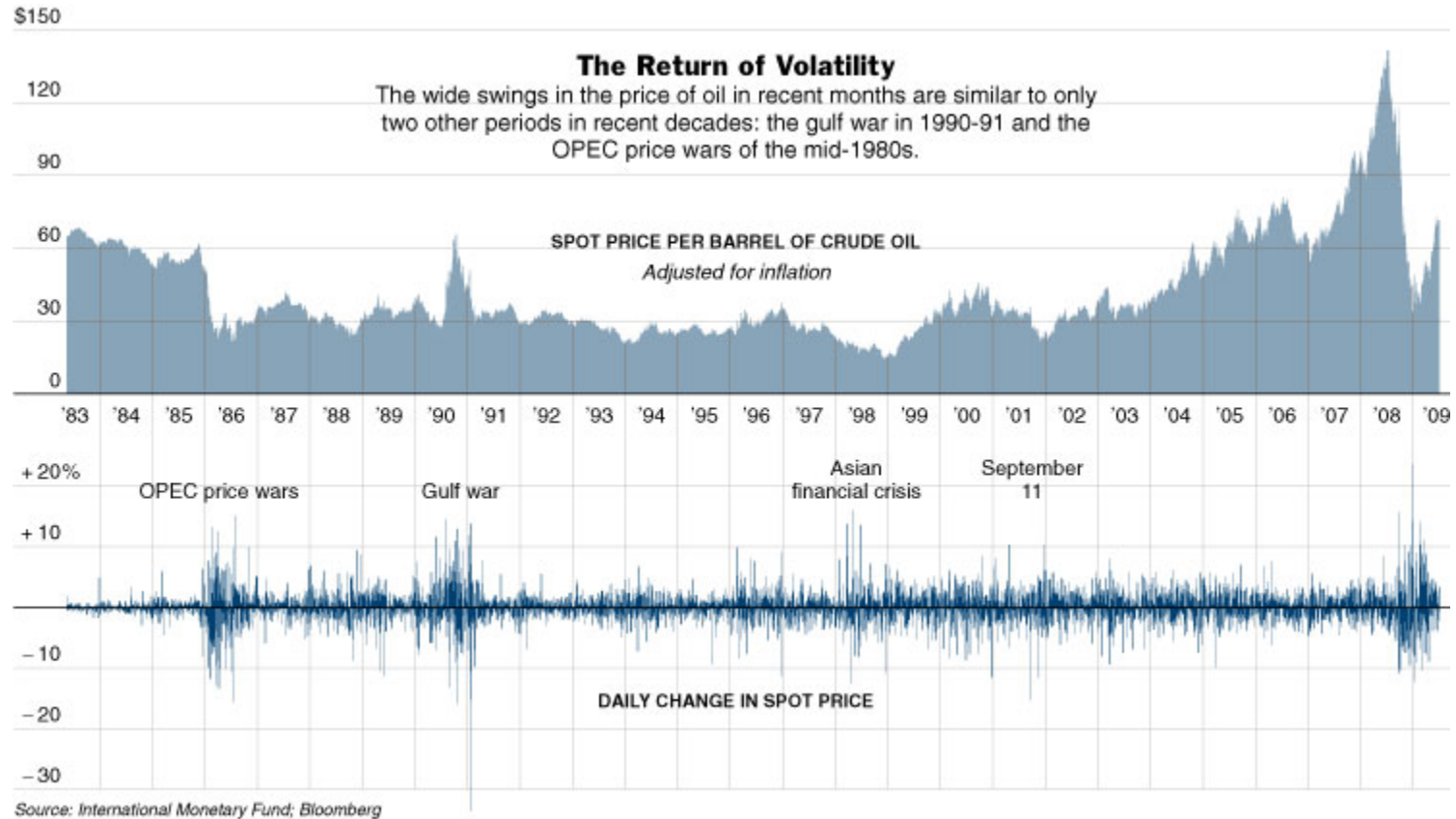
Sources:

1. Finance for All?, 2008.
2. Banamex-UNAM Survey, 2008.

I. Motivation

- Banco de México: Central Bank of Mexico.
Core function: provide currency to domestic economy.
- Forecast demand banknotes and coins.
Measure of uncertainty or volatility.
- Demand for coins is more volatile.

The New York Times



July 6th, 2009

How to formally model volatility?

Volatility

- Vague concept
 - Several definitions: e.g. IQR, s.d. , absolute returns,...
 - Higher variability at scattered times.
- EDA
 - Transformations i.e. $\ln(p_t / p_{t-1}) \approx r_t$
 - Graphical methods: time plots, boxplots,...
 - Frequency domain: dynamic spectrum,...

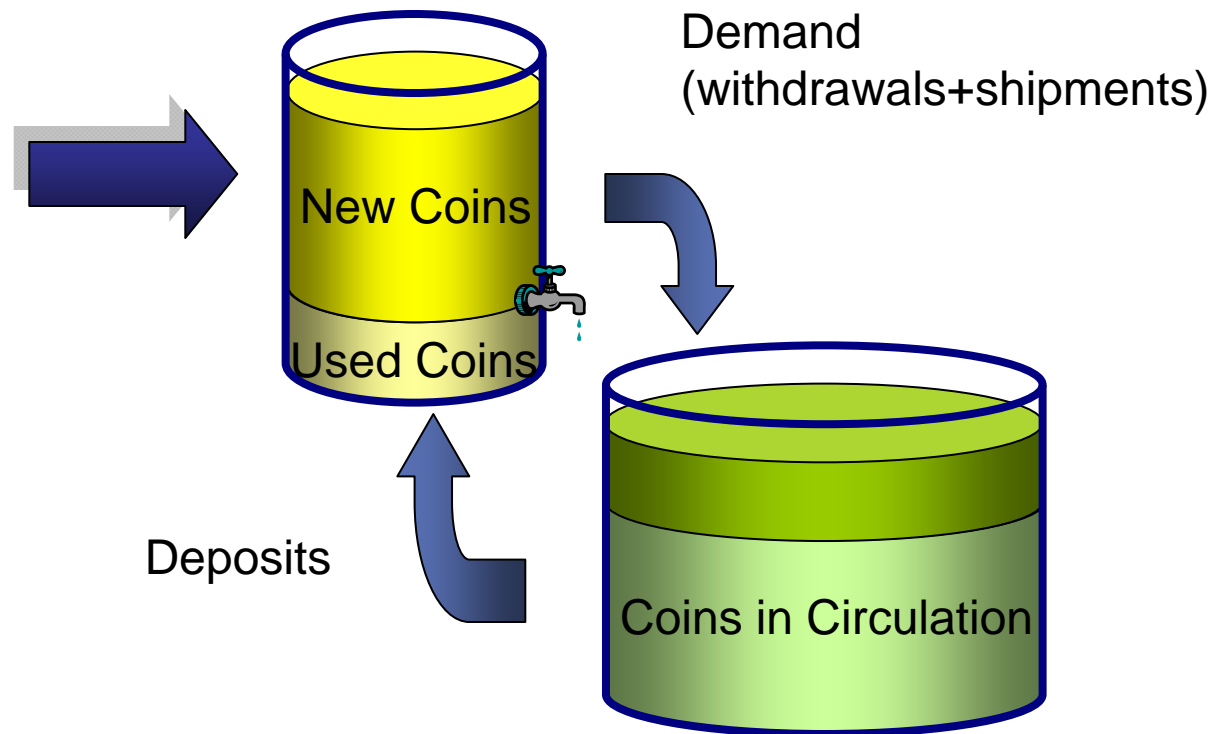
Volatility

- Formal models
 - Conditional volatility models:
ARCH, GARCH, ...
 - Stochastic volatility models
 - Non-parametric: smoothing, filtering,...

Coin distribution



Casa de Moneda
(factory)



II. Data

- Real-valued discrete-time series:
 - Daily demand for coins
 - Jan 1999 - Oct 2006
 - Business days data (400 weeks)
 - Denominations:
 - Lower: 5, 10, 20 and 50 centavos
 - Higher: 1, 2, 5, 10 pesos

- Stationarity: unchanged by time translation
Second order properties $\{Y_t\}_{t=0,\dots,T-1}$:

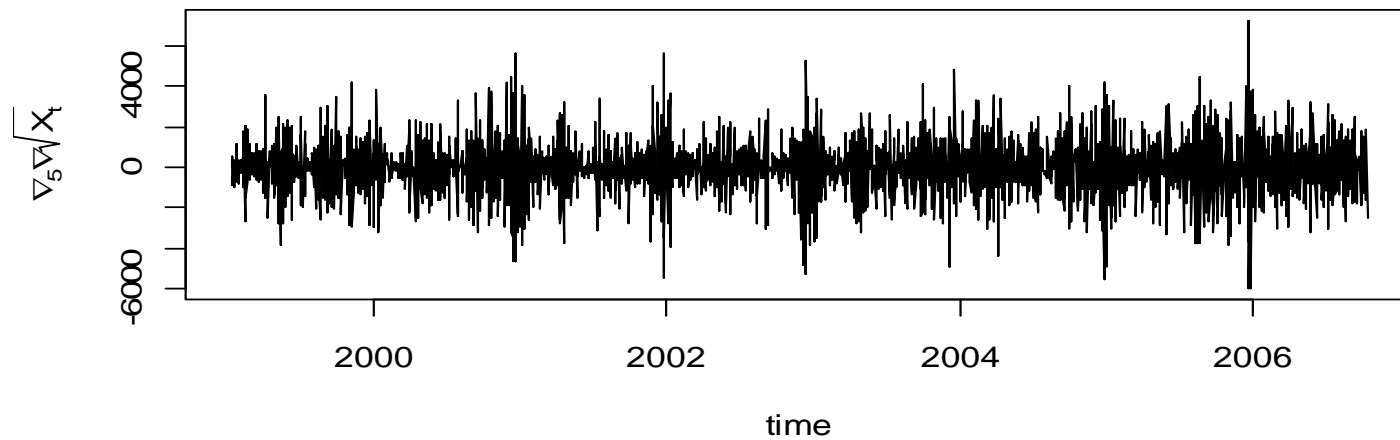
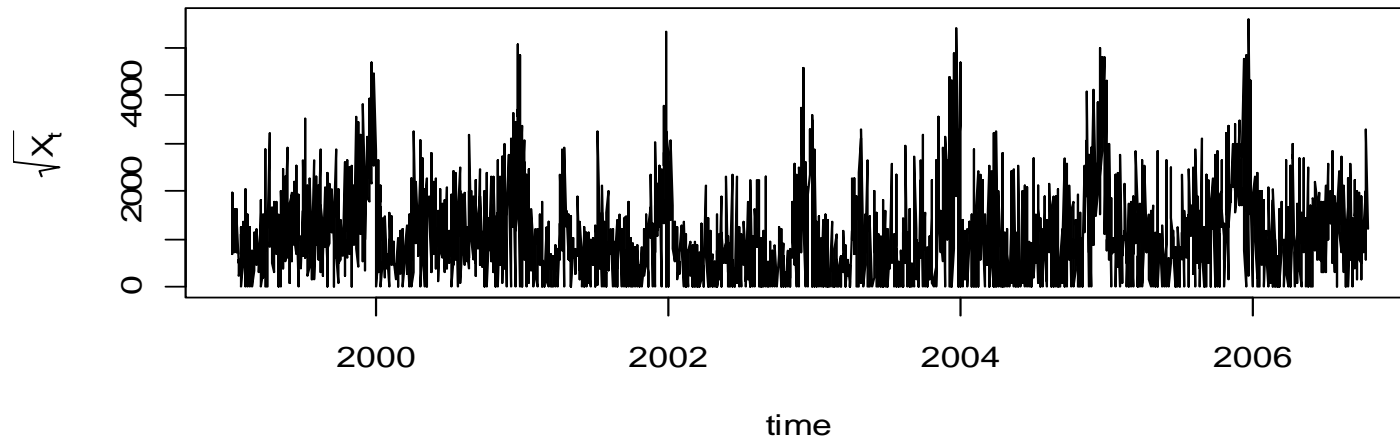
- Covariance (cumulant function order 2)

$$c_{YY}(u) = \text{cov}(Y_t, Y_{t+u}) \quad u = 0, \pm 1, \pm 2, \dots$$

- Spectral density (cumulant spectrum order 2)

$$f_{YY}(\lambda) = \frac{1}{2\pi} \sum_{u=-\infty}^{\infty} c_{YY}(u) e^{-i\lambda u} \quad -\infty \leq \lambda \leq \infty$$

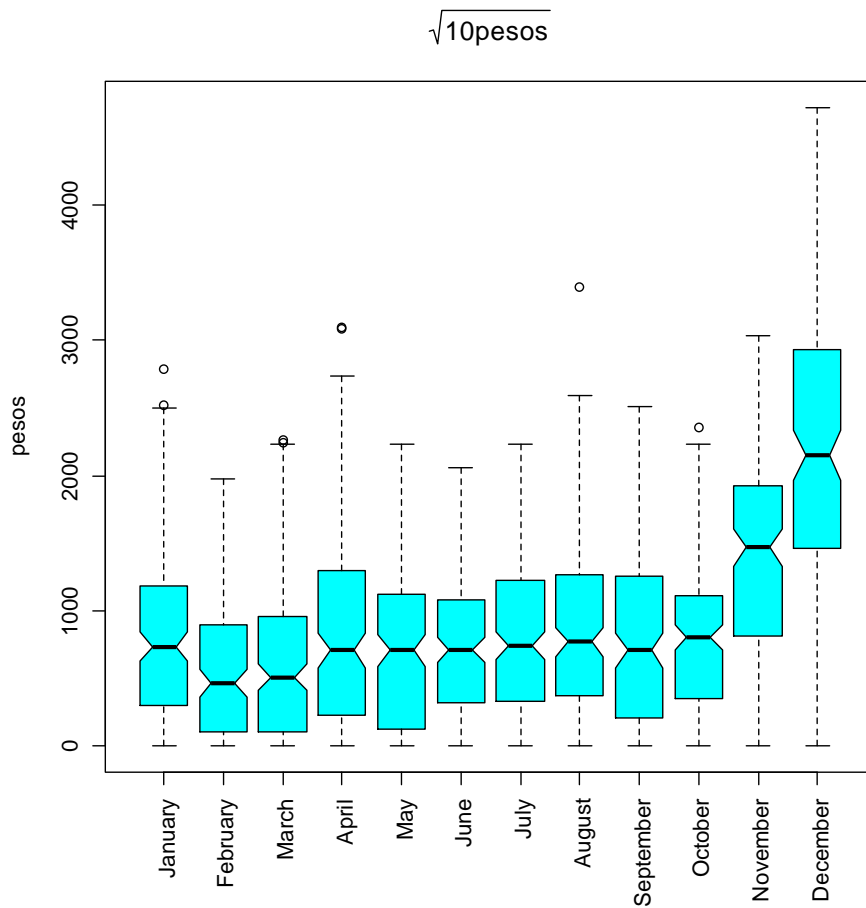
$$E_{XX}[I_{XX}(\lambda_{j:n})] \longrightarrow f_{XX}(\lambda) \quad \lambda_{j:n} = \frac{j_n}{n} \rightarrow \lambda \neq 0 \text{ as } T \rightarrow \infty$$



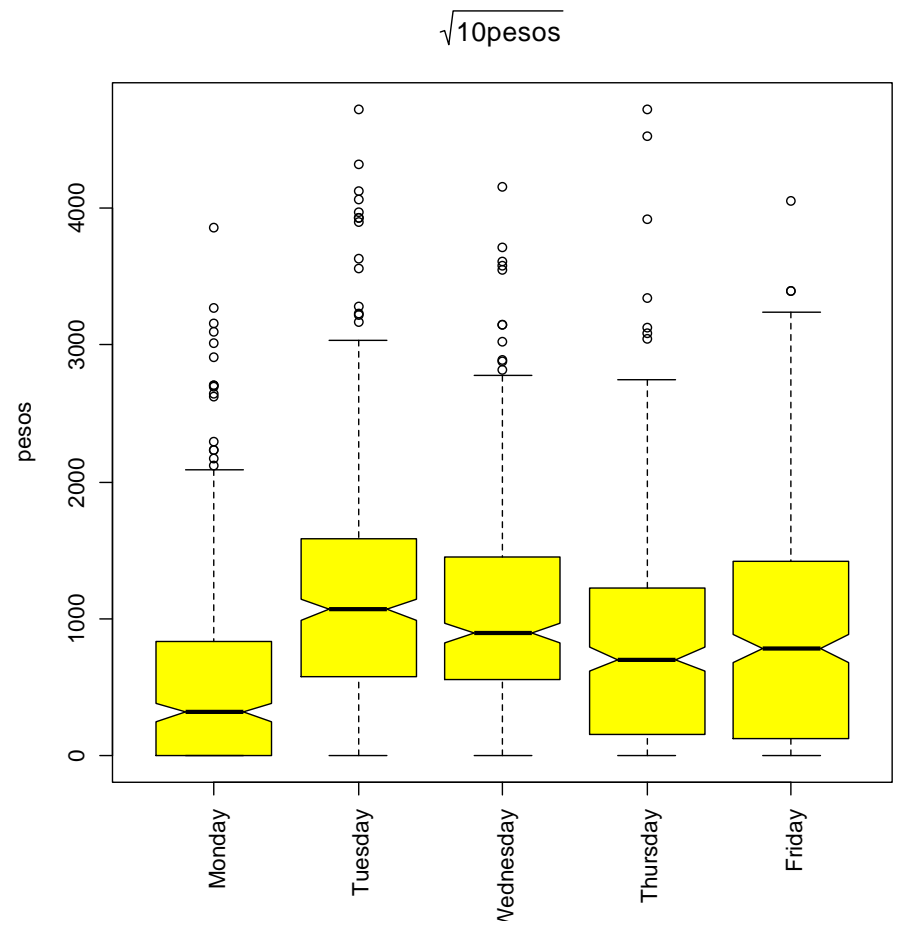
$X_t = 10$ pesos demand.
Weekly & trend effect.

$$(1 - B^5)(1 - B)\sqrt{X_t} = \nabla_5 \nabla \sqrt{X_t}$$

Seasonalities

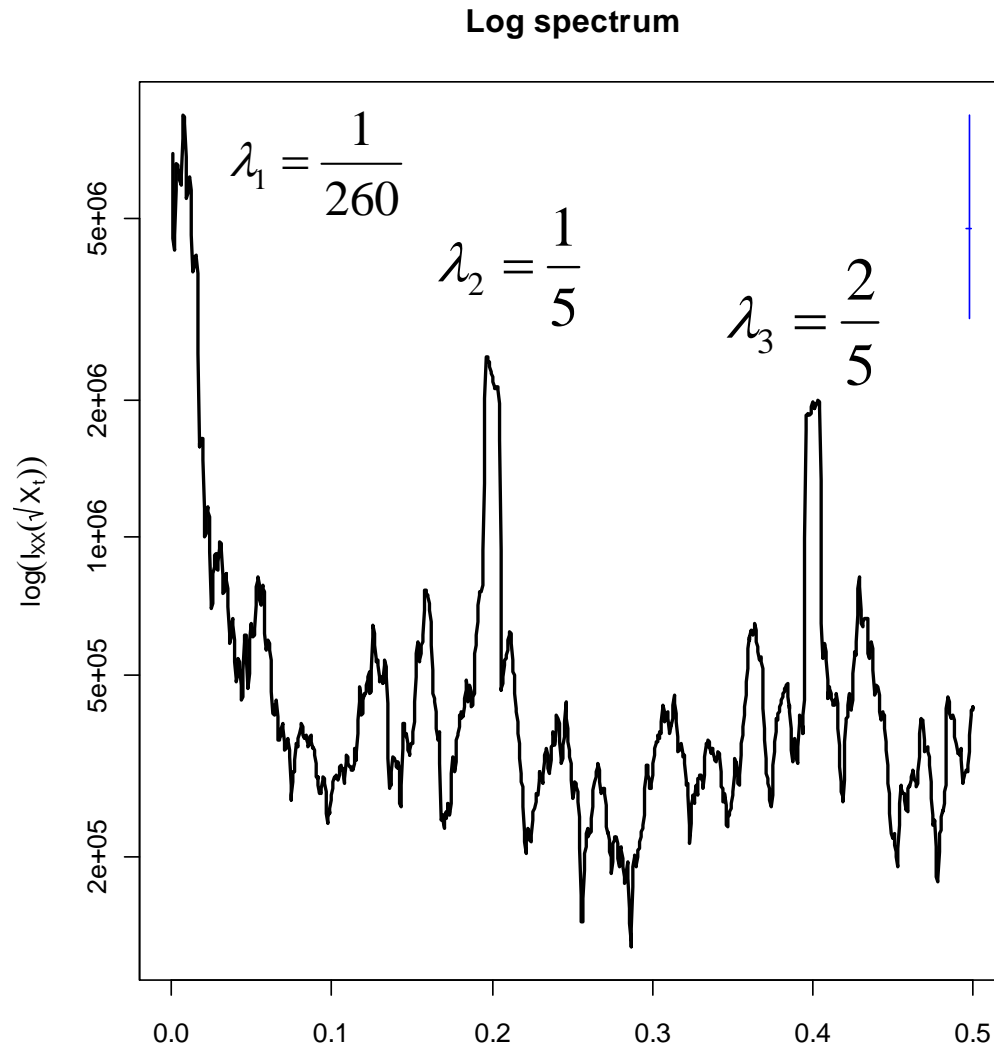


aprox 260 days



aprox 5 days

- Fourier analysis:
identify dominant frequencies.



III. Time Domain Approach

- S-ARIMAX (p,d,q) x (P,D,Q)_s

$$Y_t = \sum_{i=1}^p \phi_i Y_{t-i} + \sum_{j=1}^P \Phi_j Y_{t-sj} + \sum_{k=1}^q \theta_k \varepsilon_{t-k} + \sum_{l=1}^Q \Theta_l \varepsilon_{t-sl} + \sum_n \beta_n \gamma_n(t) + \varepsilon_t$$

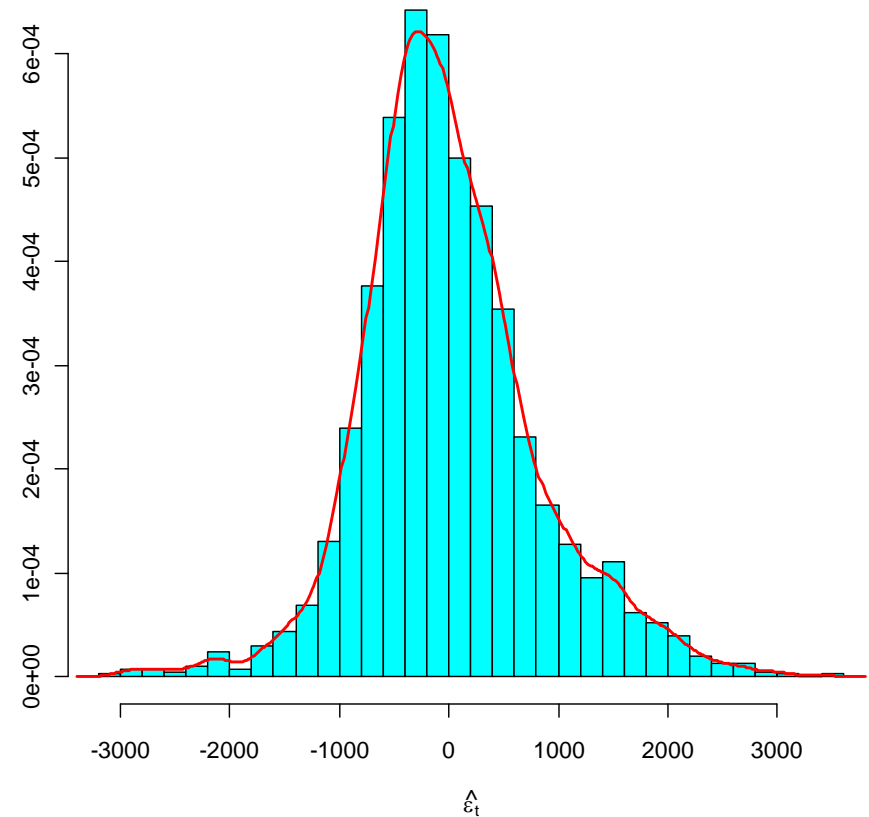
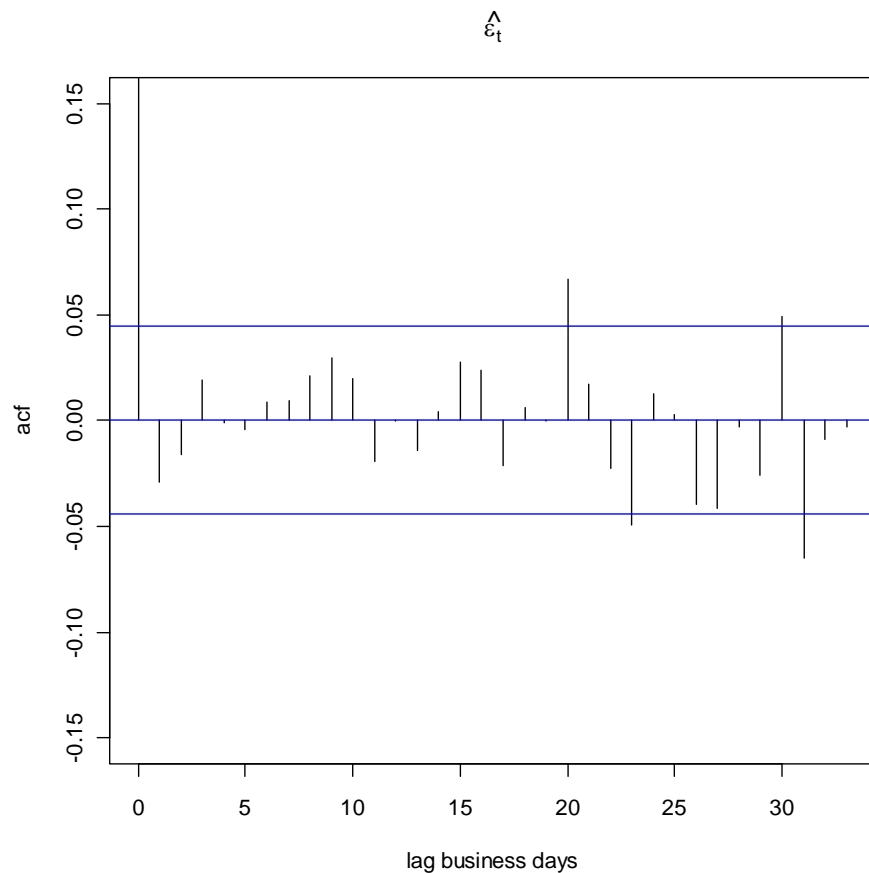
$t = 1, \dots, T-1$

$$\sum_n \beta_n \gamma_n(t) = \sum_{n=1}^2 \beta_{n1} \sin\left(\frac{2\pi nt}{5}\right) + \beta_{n1} \cos\left(\frac{2\pi nt}{5}\right)$$

$$Y_t = \nabla_5 \nabla X_t$$

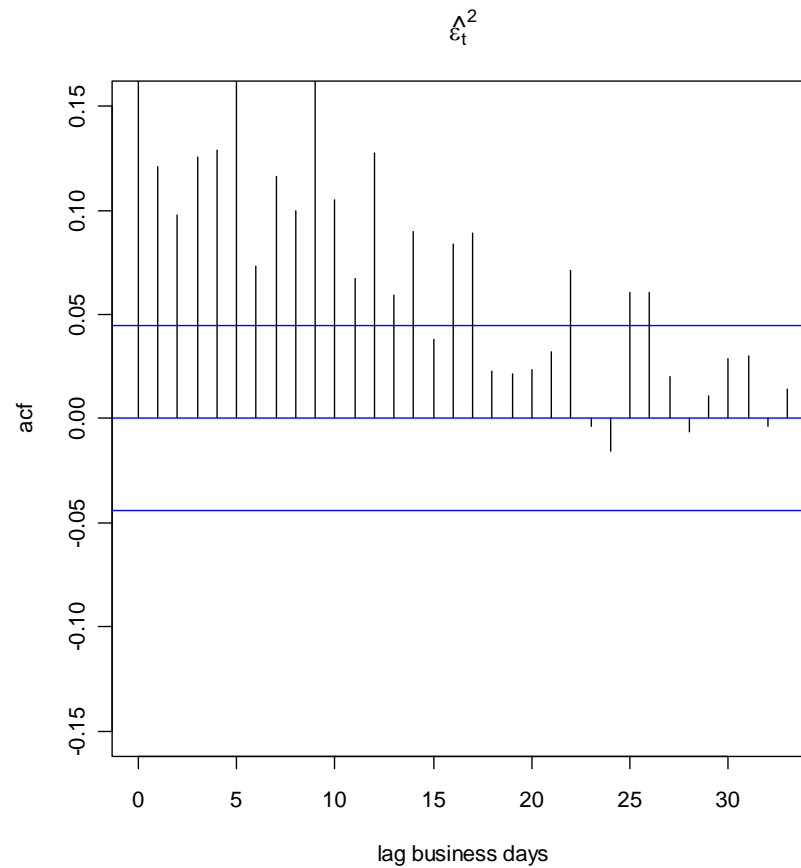
Results: second order fit

- $\hat{\varepsilon}_t$: residuals S-ARIMAX(0,1,1)x(1,1,1)₅

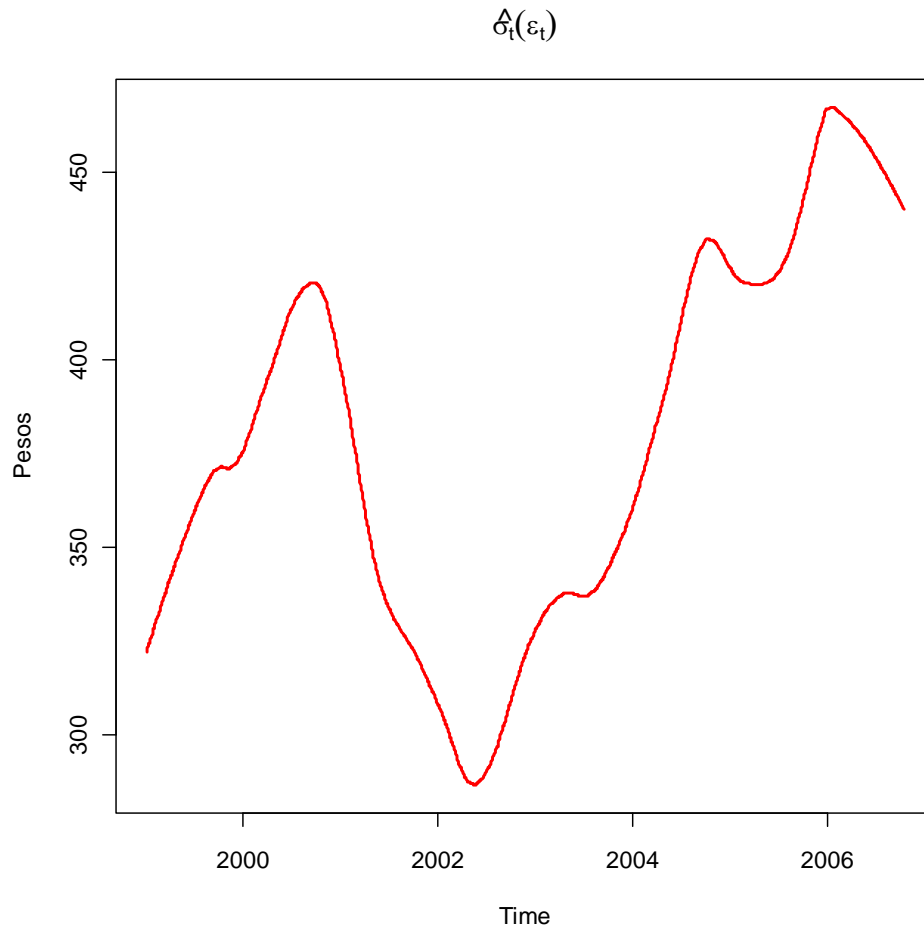


Higher-order fit?

- $\hat{\varepsilon}_t^2$ residuals squared



Trend?



Smoothed Running Absolute Deviation (SRAD)

$$\hat{\sigma}(\hat{\varepsilon}_t) \approx \text{lowess}(|\hat{\varepsilon}_t - \text{med}_{j \in N(\hat{\varepsilon}_t)}(\hat{\varepsilon}_j)|)$$

$$N(\hat{\varepsilon}_t) = \{i-2, i-1, i, i+1, i+2\}$$

GARCH(m,s)

$$X_t = \sqrt{h_t} w_t$$

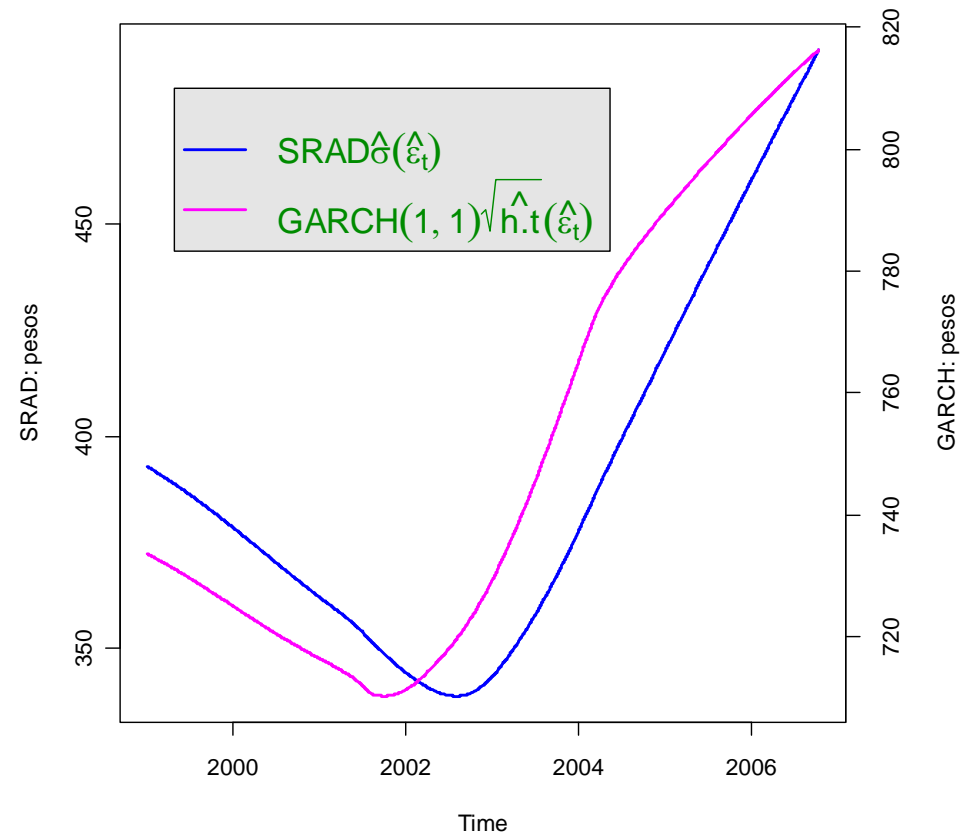
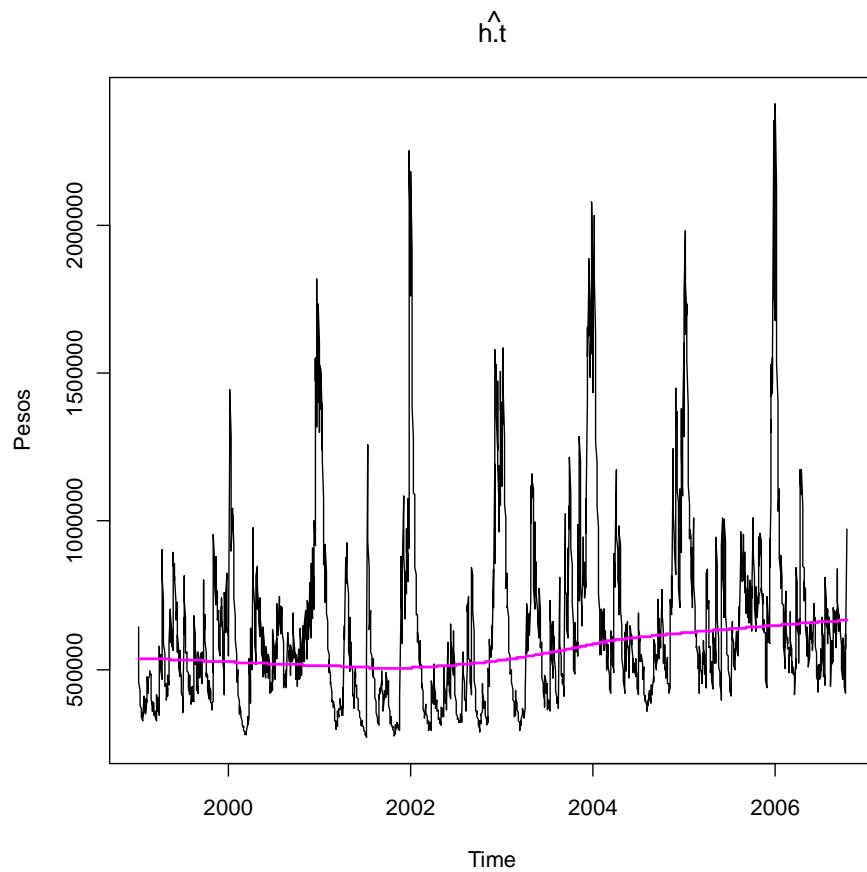
$$h_t = c + \sum_{i=1}^s a_i h_{t-i} + \sum_{j=1}^m d_j h_{t-i} w_{t-i}^2$$

$$h_t = \text{Var}(X_t | H_t)$$

w_t : Gaussian white noise with $E(w_t)=0$ and $\text{Var}(w_t)=1$

Results: GARCH(1,1)

Estimated conditional variance \hat{h}_t



IV. Frequency Domain Approach

3rd order properties

- Cumulant function of order 3

$$c_{YYY} = c_{YYY}(u_1, u_2) = \text{cum}\{ Y_t, Y_{t+u_1}, Y_{t+u_2} \}$$

- Bispectrum

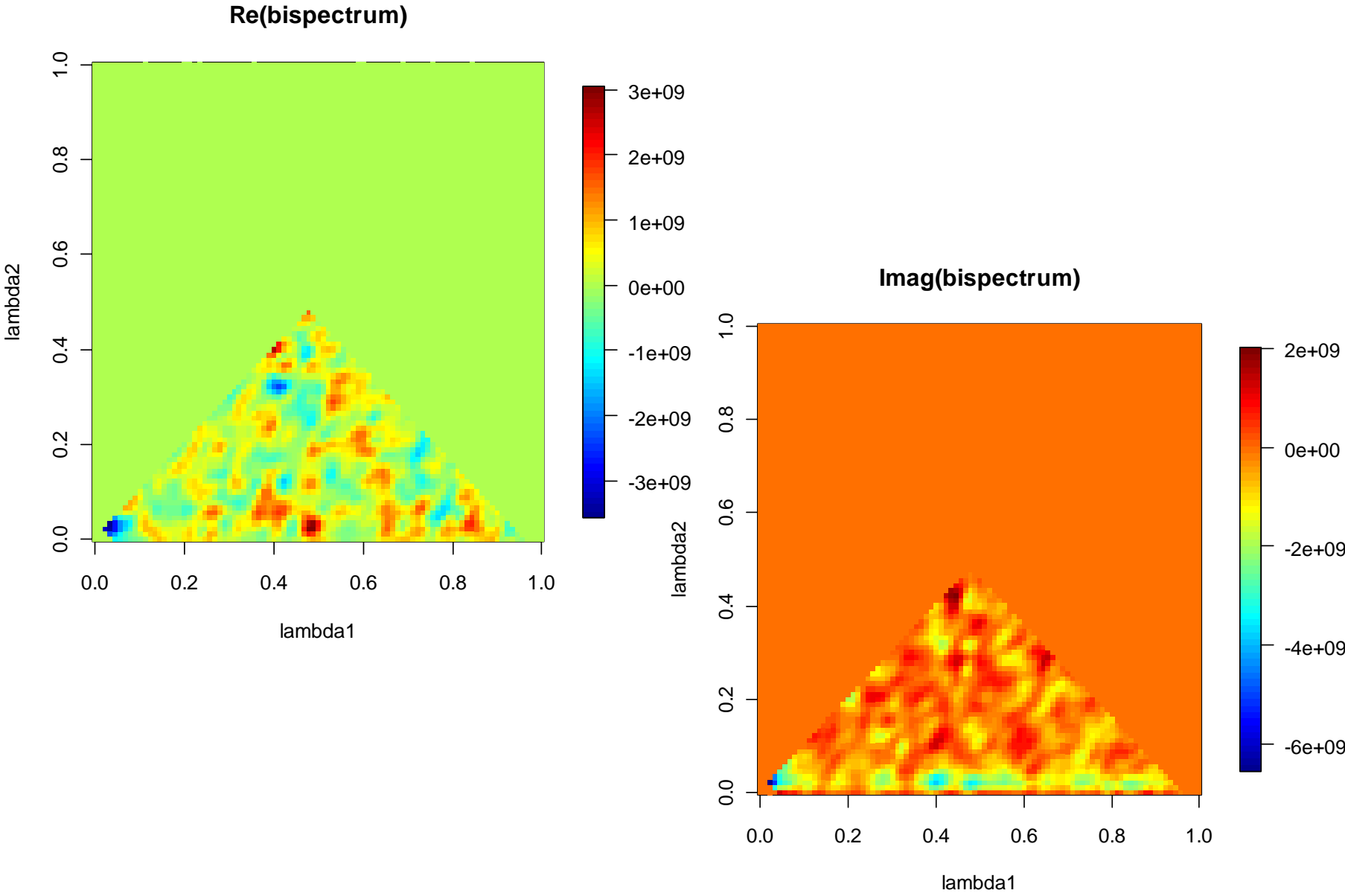
$$f_{YYY}(\lambda_1, \lambda_2) = \frac{1}{(2\pi)^2} \sum_{u_1=-\infty}^{\infty} \sum_{u_2=-\infty}^{\infty} c_{YYY}(u_1, u_2) e^{-i(\lambda_1 u_1 + \lambda_2 u_2)}$$

$$I_{YYY}(\lambda_1, \lambda_2) = \frac{1}{(2\pi)^2} \frac{1}{T} d(\lambda_1) d(\lambda_2) \overline{d(\lambda_1 + \lambda_2)} \quad -\infty \leq \lambda \leq \infty$$

GARCH(1,1)

- GARCH (1,1) is a particular stationary state space model.
- Frequency domain approach:
2nd order spectrum (power spectrum) and
3rd order spectrum (bispectrum).

Bispectrum of the S-ARIMAX residuals



Estimation

- S-ARIMAX spectral domain

Spectral likelihood function: under the assumption of Gaussian data

$$\Psi = (\phi_i, \dots, \theta_j, \dots, \beta_r, \sigma_\varepsilon^2) \max \prod_{0 < s < T/2} f_{YY} \left(\frac{2\pi s}{T}, \Psi \right)^{-1} \exp \left\{ -I_{YY} \left(\frac{2\pi s}{T} \right) f_{YY} \left(\frac{2\pi s}{T}, \Psi \right)^{-1} \right\}$$

Whittle, 1953

- GARCH(m,s) spectral estimation

Approximation to the likelihood:

2nd (f_{YY}) and 3rd-order spectral estimates (f_{YYY}).

$$\frac{\sum_s \left(I_{YY}^T - f_{YY} \left(\frac{2\pi s}{T}, \Psi \right) \right) \frac{\partial f_{YY}}{\partial \Psi_i}}{f_{YY}^2 \left(\frac{2\pi s}{T} \right)} + \frac{\frac{2\pi}{T} \sum_r \sum_s \left(I_{YYY}^T - f_{YYY} \left(\frac{2\pi s}{T}, \frac{2\pi r}{T}, \Psi \right) \right) \frac{\partial f_{YYY}}{\partial \Psi_i}}{f_{YY} \left(\frac{2\pi s}{T} \right) f_{YY} \left(\frac{2\pi r}{T} \right) f_{YY} \left(\frac{2\pi(s+r)}{T} \right)} = 0$$

Brillinger, 1986

Terdik, 1999

Summary

- The demand of 10 pesos coins
 - S-ARIMAX captures linear properties
 - requires a GARCH model
- GARCH leads to non-Gaussian, non-linear
- There are time domain and frequency domain approaches.
- These are being investigated
- Can be extended to incorporate seasonal volatility.

References

- Banamex-UNAM, 2008. Primera encuesta sobre cultura financiera en México.
- D. Brillinger, 1994. *Some basic aspects and uses of higher-order spectra.*
- S. Poon and C. Granger, 2003. *Forecasting Volatility in Financial Markets: A Review.*
- R. Shumway and D. Stoffer, 2006. *Time Series Analysis and Its Applications. With R Examples.*
- G. Terdik, 1995. *Bilinear Stochastic Models and Related Problems of Nonlinear Time Series Analysis. A Frequency Domain Approach.*
- World Bank Policy Research Report, 2008. *Finance for All? Policies and Pitfalls in expanding Access.*
- *R libraries: timsac, fGarch.*

Thank you

Fourier Analysis of stationary processes

- Fourier transform: linear and translation invariant.
- Linear filter easily described for Fourier transform function.
- Simpler large sample statistical properties of the transforms.
- FFT algorithms.
- Estimation of finite parameter models.

Identifying volatility

Suppose have stationary zero mean series $\{X(t)\}$ with

$$\begin{aligned} \text{autocov } c_{XX}(u) &= \sigma^2 & u = 0 \\ &= 0 & u \neq 0 \end{aligned}$$

Let $Y(t) = X(t)^2$

$$\begin{aligned} \text{autocov } c_{YY}(u) &= c_{XXXX}(0, u, 0) & u \neq 0 \\ &= 2\sigma^4 + c_{XXXX}(0, 0, 0) & u = 0 \end{aligned}$$

with c_{XXXX} the cumulant function of $\{X\}$ of order 4

For a Gaussian series the 4th order cumulants are 0 so a type of non gaussianity is basic to the phenomenon.



Parametric model

GARCH(m,s)

Mean

$$y_t = \phi_0 + \sum_{i=1}^p \phi_i y_{t-i} + \sum_{i=1}^q \theta_i a_{t-i} + a_t \quad |\phi_i| < 1$$
$$a_t = \sigma_t \varepsilon_t \quad E(\varepsilon_t) = 0, \text{Var}(\varepsilon_t) = 1, \varepsilon_t \text{ iid}$$

Conditional Var

$$\text{Var}(y_t | H_{t-1}) = \sigma_t^2 = \alpha_0 + \sum_{i=1}^s \kappa_i \sigma_{t-i}^2 + \sum_{j=1}^m \alpha_j a_{t-j}^2$$

H_{t-1} : information before time t

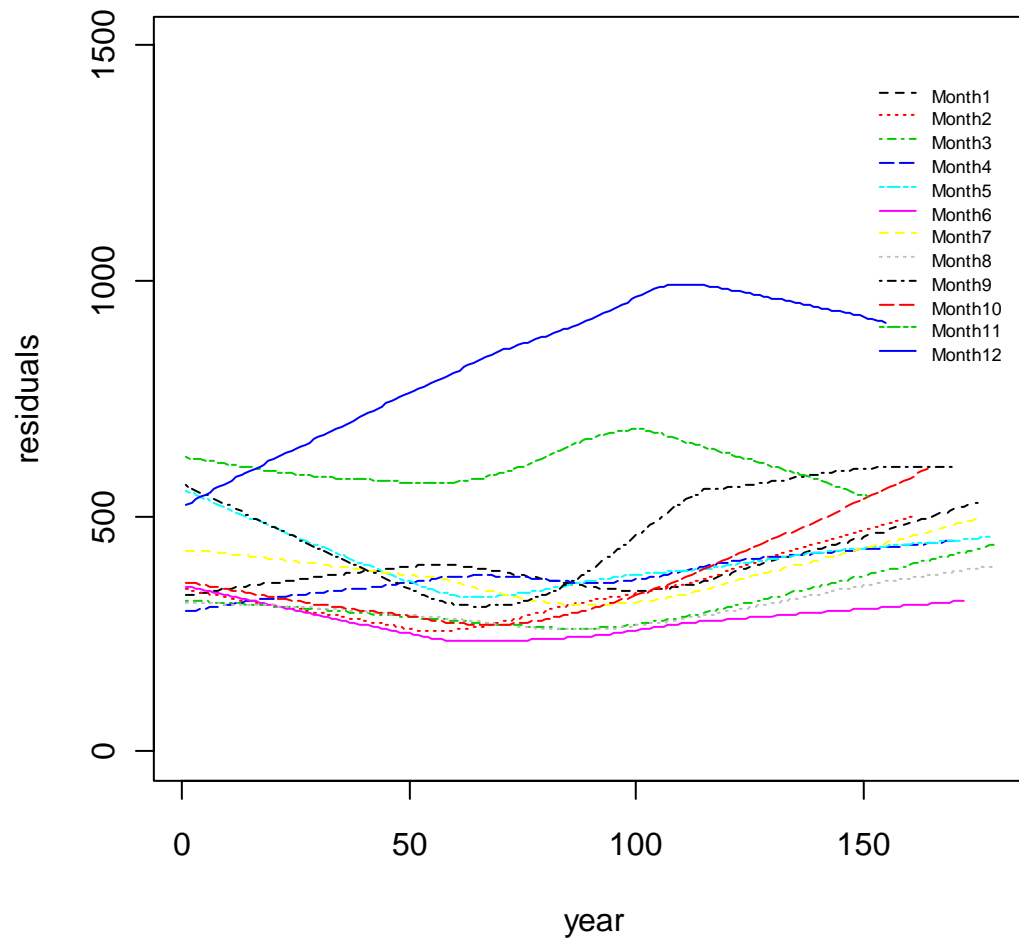
$$\alpha_0 > 0 \quad \text{and} \quad \alpha_i \geq 0 \quad \text{for } i > 0$$

$$\kappa_j \geq 0 \quad \text{and} \quad \sum_{i=1}^{\max(m,s)} (\alpha_i + \kappa_i) < 1$$

Conditions for stationarity

Seasonal Volatility?

Volatility by month 10 pesos



IH1

Include confidence intervals per month

Irma Hernandez, 7/27/2009