FINAL EXAM, STAT 134 SECTION 3

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Instructions: Please PRINT your name and SID # legibly below. You will have 50 minutes to complete 4 problems. In addition, please print your name at the top of EACH page.

There is a space for answers immediately following each question. Please record your answer in this space.

Name:	
SID#:	

Pr #:	Score:	Pr #:	Score:
1		2	
3		4	

Total:	

(1) Let $S_n = X_1 + \cdots X_n$ where the random variables X_i are independent, identically distributed Bernoulli's with parameter p. Let $m < n$.

(a) For each value of k compute, if possible,

$$\mathbb{E}\left[S_n | S_m = k\right].$$

For those values where a computation is not possible say why not.

(b) Under the previous assumptions, give a formula for

$$\mathbb{E}\left[S_n | S_m\right].$$

(c) Repeat (a) for

$$\mathbb{E}\left[S_m | S_n = k\right].$$

(d) Give a formula for

$$\mathbb{E}\left[S_m | S_n\right].$$

(2) Let X_1, X_2, and X_3 be independent normal random variables with respective means $1, 4, 7$ and variances $1, 1, 1$.

(a) Compute $\mathbb{E}\left[X_1 + X_2 + X_3\right]$

(b) Compute $Var\left(X_1 + X_2 + X_3\right)$

(c) Compute $\mathbb{P}\left(X_1 + X_2 + X_3 > 13\right)$ to two decimals (i.e.: .55).

(d) Use scale transformations to express the χ^2 distribution with 3 degrees of freedom in terms of X_1, X_2, X_3.

(3) Suppose that you have 3 bags mixed with black and white marbles. You draw one marble, randomly selecting the bag to choose from. You draw from Bag i with probability $\frac{i}{6}$. Assume that each bag has unknown proportions of black marbles p_i (the label i corresponding to the bag in question).In terms of the p_i's compute each of the following:

(a)
$$\mathbb{P}(\text{ Marble came from Bag } i \mid \text{Marble is white})$$

(b) Now suppose you pick a bag at random and draw 10 marbles with replacement from this bag, compute
$$\mathbb{E}\left[N_w\right]$$
where N_w is the number of white marbles after the ten draws.

(c) Under the same process as in Part (b) compute
$$\mathbb{E}\left[N_{w,1}\right]$$
with $N_{w,1}$ the number of white marbles which came from Box 1.

(4) Let T be an exponential distribution with rate λ. Define the *integer* valued random variable $int(T)$ to be the greatest integer less than or equal to T.

(a) Compute the probability mass function for $int(T)$.

(b) Use properties of exponents to identify the discrete distribution with one we have studied this semester.

(c) Let T_1 and T_2 be independent exponentials with rate λ. Compute the probability mass function for $int(T_1 + T_2)$ with *int* defined analogously to the previous problem.

(d) Is this the same as $int(T_1) + int(T_2)$?

(5) Let X, Y be a pair of random variables with joint distribution which is uniform on the triangle R where

$$R := \{(x, y) : x + y \leq 1, x, y \geq 0\}.$$

(a) For each $y \in \mathbb{R}$, find the conditional density for X, given that $Y = y$. Treat your answer carefully depending on the particular value of y under consideration.

(b) Compute the marginal density for Y.

(c) Compute $\mathbb{E}[X|Y]$.

(d) Compute $Var(X|Y = y)$ paying careful to the particular value of y under consideration.

(6) Let T be an exponential random variable with rate λ.

(a) Compute the distribution for

$$Y = e^{-\lambda T}.$$

(b) Compute the distribution for

$$Z = \sqrt{T}.$$

(c) Find a value of λ for which the density for Z has the standard form of the Rayleigh Distribution.