

Volatility

Volatility: important topic for time series, particularly financial series.

Vague concept, capable of formalization in variety of ways.

Persistent notion: local variability, values are changing a lot.

Financial world: "A statistical measure of the dispersion of returns for a given security or market index.

Measured using standard deviation (variance) between returns from security or market index.

Higher the volatility, the riskier the security."

Study can lead to better assessment of risk, to better forecasting, and to checking that a process control.

Many risk analysis questions involve some form of variability.

In insurance safety loaded pure risk premium can take the form

$$\lambda_1 P(t) + \lambda_2 \sigma(t) + \lambda_3 \sigma(t)^2$$

where the λ 's are weights, $P(t)$ is the fair premium and $\sigma(t)$, or $\sigma(t)^2$ is the volatility at time t .

Another measure of financial risk: Value at Risk (VAR), defined as the maximum expected loss over a specified time period with a given confidence level.

Losses in the time horizon will exceed the VAR only with

prespecified probability α .

Estimates of something like $\sigma(t)$ are required in its computation.

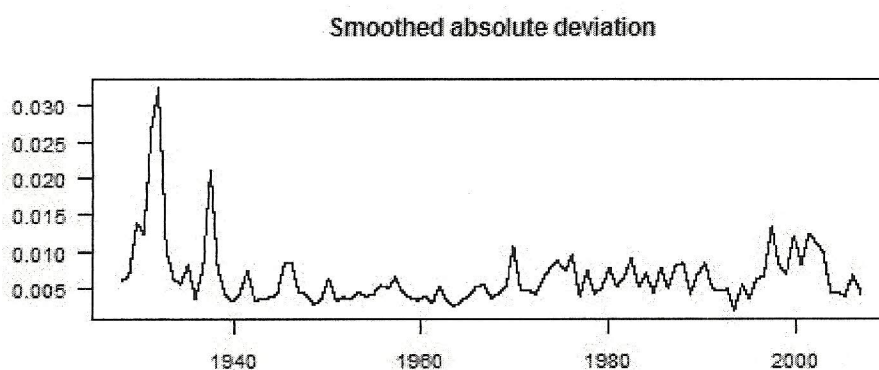
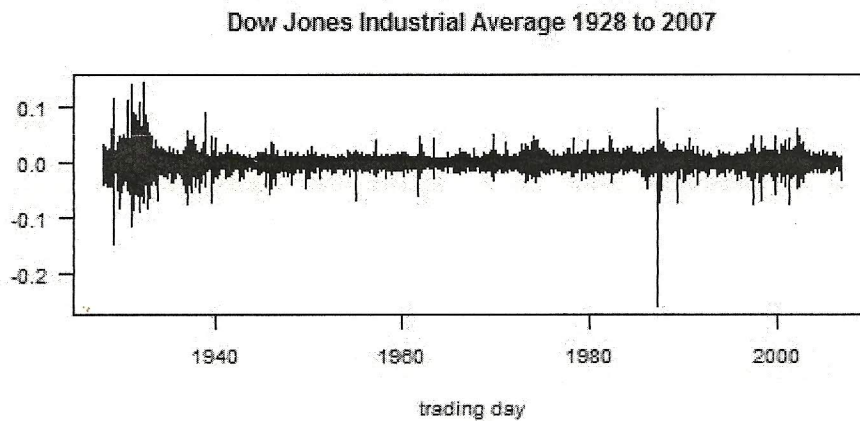
Example. Dow Jones Industrial Average.

Based on prices of 30 stocks

Return, $R(t) = \log Y(t)/Y(t-1)$, studied for volatility.

Naïve estimates of volatility:

$$\text{ave}\{ \sum |Y(s) - Y(s-1)| \} \quad \text{and} \quad \text{ave}\{ \sum [Y(s) - Y(s-1)]^2 \}$$



Volatility of upper figure confirmed in the lower.

Periods of high volatility: crash of 1929, Black Monday of 1987, 9/11 period of 2001.

(1932, 1937, 1940, 1962, 1987)

GARCH model

$$R(t) = \mu(t) + \sigma(t)\varepsilon(t)$$

$$\sigma(t)^2 = \alpha_0 + \sum \alpha_i \varepsilon(t-i)^2 + \sum \beta_j \sigma(t-j)^2, \text{ volatility}$$

α 's and β 's non-negative.

Parameters estimated, then estimate of $\sigma(t)$ can be fed into risk computations.