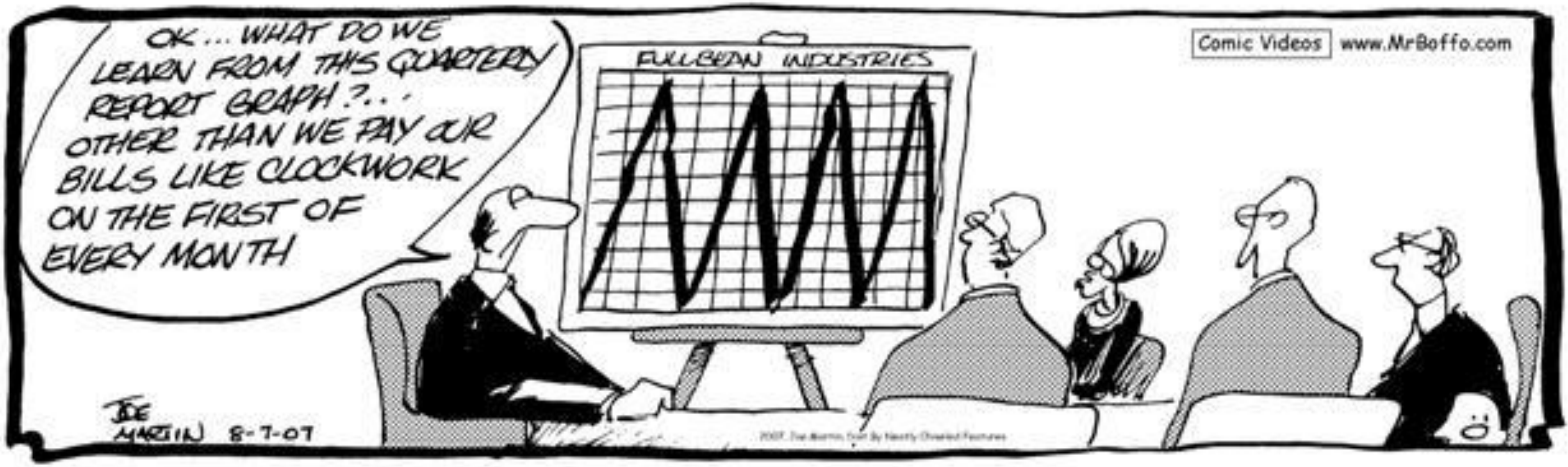

OK... WHAT DO WE LEARN FROM THIS QUARTERLY REPORT GRAPH ?... OTHER THAN WE PAY OUR BILLS LIKE CLOCKWORK ON THE FIRST OF EVERY MONTH
FULLBEAN INDUSTRIES
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JOE MARTIN 8-7-07

Extending the VolatilityConcept to Point Processes

David R. Brillinger

Statistics Department

University of California

Berkeley, CA

brill@stat.berkeley.edu

www.stat.berkeley.edu/~brill

$2\pi \neq 1$

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Volatility sweeps global markets

US stock markets have dropped sharply, extending a global share sell-off amid fears about the effect of higher interest rates on the world economy.

There are concerns that the drop in share prices could be severe

There are concerns that higher rates will hit corporate profits and takeover deals, and dent consumer spending.

European markets were also jittery, with London's share index closing down for a fourth day and ending at its lowest level since the middle of March.

Analysts have warned that markets could remain volatile for a number of weeks.

"I think you've got bargain hunters out there for sure and I think you've got some people who are still scared," said Randy Frederic of Charles Schwab & Co.

"We're seeing the convergence of a whole host of sort of

MARKET DATA - 14:35 UK

FTSE 100	6215.2 ▼	-36.00
Dax	7451.7 ▼	-57.28
Cac 40	5644.0 ▼	-31.09
Dow Jones	13265.5 ▼	-208.10
Nasdaq	2562.2 ▼	-37.10
S&P 500	1459.0 ▼	-23.71
BBC Global 30	5723.3 ▼	-47.66

OPEN Marketwatch ticker

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Introduction

Is there a useful extension of the concept of volatility to point processes?

A number of data analyses will be presented

Why bother with an extension to point processes?

a) Perhaps will learn more about time series case

b) Pps are an interesting data type

c) Pps are building blocks

d) Volatility often considered risk measure for time series.

27 July Guardian.

"Down nearly 60 points at one stage, the FTSE recovered and put on the same amount again. But by the close it had slipped back, down 36.0 points."

"inject billions into the banking system"

Volatility

- When is something volatile?
  - When values shifting/changing a lot

Vague concept

- Can be formalized in various ways
- There are empirical formulas as well as models

Merrill Lynch.

"Volatility.

A measure of the fluctuation in the market price of the underlying security.

Mathematically, volatility is the annualized standard deviation of returns.

A *- Low;* ***B*** *- Medium; and* ***C*** *- High."*

Financial time series.

P_t price at time t

"Return" data, $Y_t = (P_t - P_{t-1})/P_{t-1}$

Empirical formula

Realized volatility

$$\text{mean}\{|Y_s - Y_{s-1|}^p \mid s \text{ near } t\}, \quad p = 1 \text{ or } 2$$

Model based formula. GARCH

$Y_t = \mu_t + \sigma_t \varepsilon_t$, ε zero mean, unit variance, t discrete

$\sigma_t^2 = \alpha_0 + \sum \alpha_i [Y_{t-i} - \mu_{t-i}]^2 + \sum \beta_j \sigma_{t-j}^2$ α's, β's > 0

Volatility σ_t^2

For μ_s, σ_s smooth

$\text{mean}\{[Y_s - Y_{s-1}]^2 \mid s \text{ near } t\} \sim \sigma_t^2 (\varepsilon_t - \varepsilon_{t-1})^2$

Crossings based.

$$E\{[Y(t) - Y(t-h)]^2\} = 2[c(0) - c(h)] \approx -2c''(0)h^2$$

Recognize as

$$2c(0)\,\pi^2\,[E(\#\{\text{crossings of mean}\})]^2$$

for stationary normal

Consider

$$s\,\#\{\text{crossings of mean} \mid \text{near } t\}$$

as volatility measure

Example. Standard & Poors 500.

Weighted average of prices of 500 large companies in US stock market

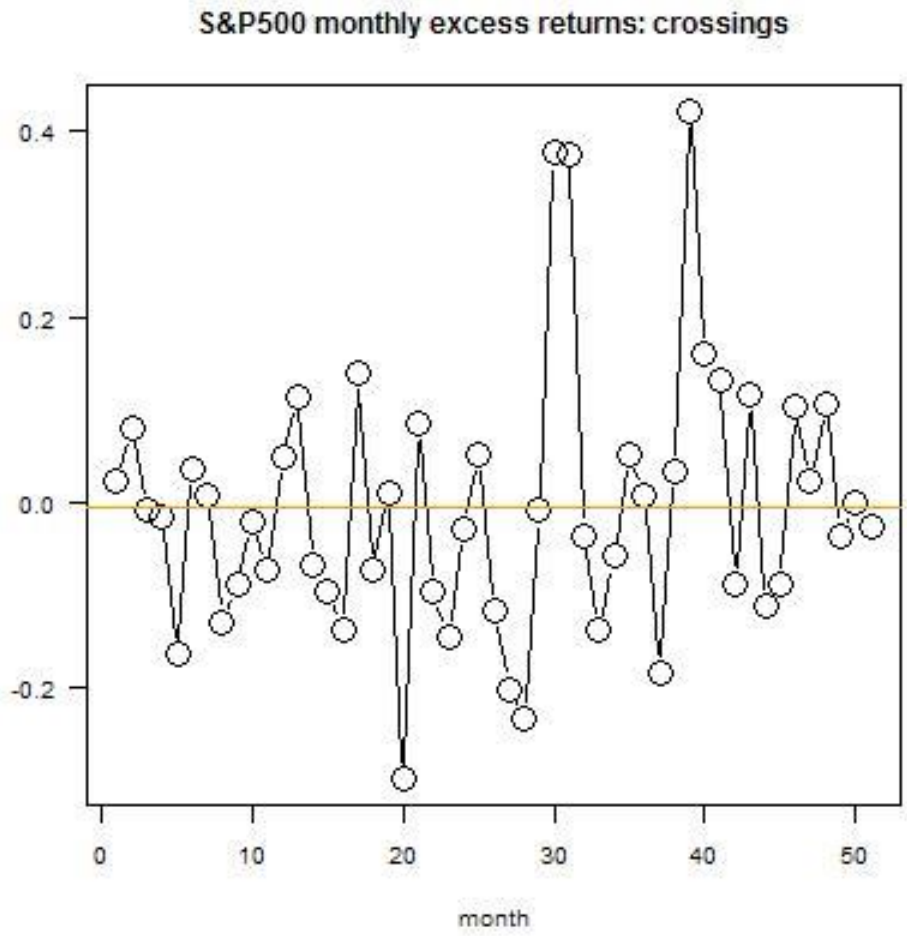
Events

Great Crash Nov 1929

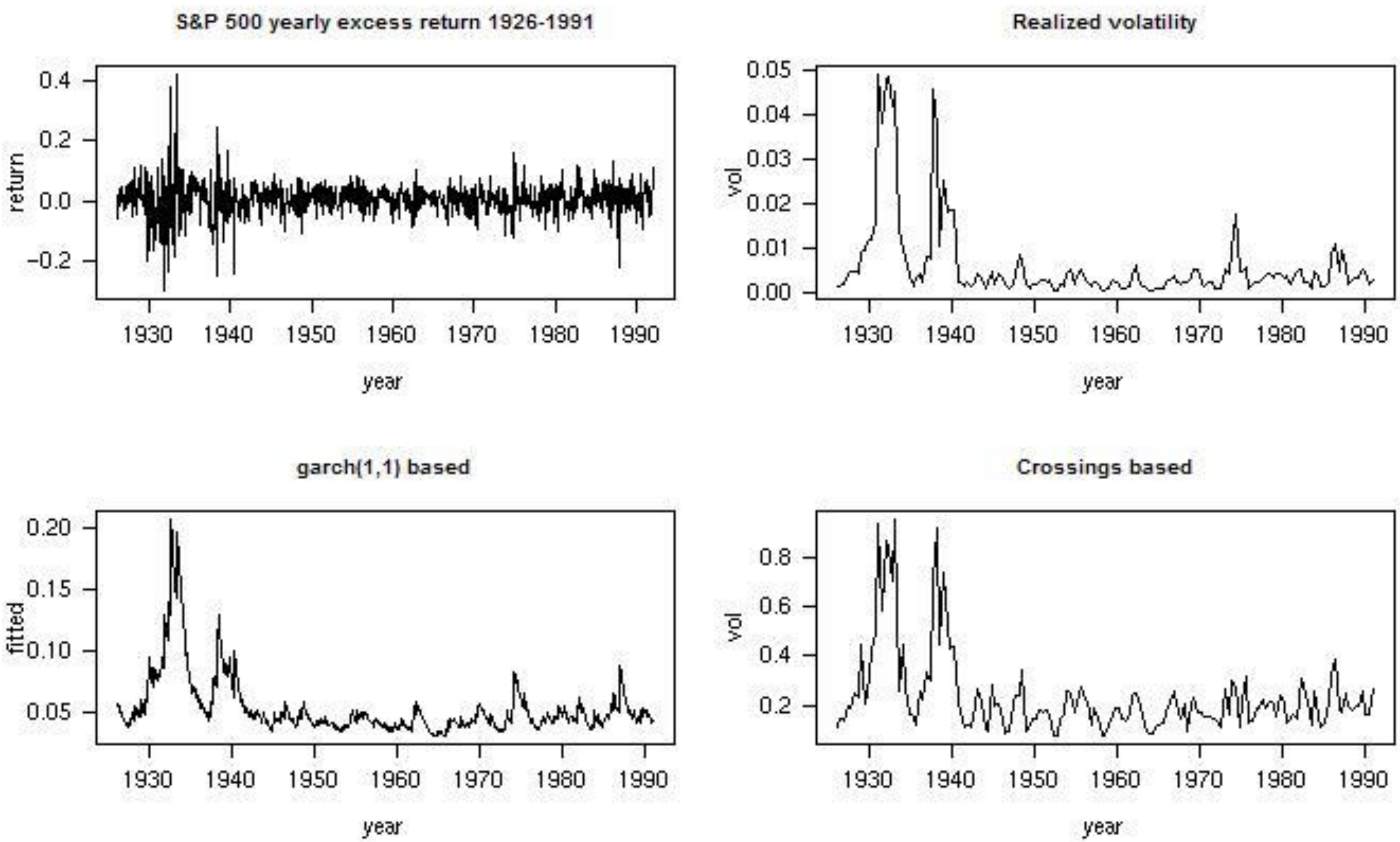
Asian Flu (Black Monday) Oct 1997

Other “crashes”: 1932, 1937, 1940, 1962, 1974, 1987

Zero crossings



S&P 500: realized volatility, model based, crossing based



n= 792

Tsay (2002)

Point process case.

locations along line: $\tau_1 < \tau_2 < \tau_3 < \tau_4 < \dots$

$N(t) = \#\{\tau_j \leq t\}$

Intervals/interarrivals $X_j = \tau_{j+1} - \tau_j$

Stochastic point process.

Probabilities defined

Characteristics: rate, autointensity, covariance density, conditional intensity, …

E.g. Poisson, doubly stochastic Poisson

0-1 valued time series.

Z_t = 0 or 1

Realized volatility

ave{ $[Z_s - Z_{s-1}]^2$ | s near t }

Connection to zero crossings.

Z_t= sgn(Y_t), {Y_t} ordinary t.s.

$\Sigma\ [Z_s - Z_{s-1}]^2$ = #{zero crossings}

Connecting pp and 0-1 series. Algebra

$T_j = <\tau_j/h>$ $<.>$ nearest integer, embed in 0's

h small enough so no ties

$Y(t) = N(t+h) - N(t) = \int_t^{t+h} dN(u)$

Stationary case

$E\{Y(t)\} = p_N \, h$

$\mathrm{cov}\{Y(t+u),Y(t)\} = \int_{t+u}^{t+u+h} \int_t^{t+h} \mathrm{cov}\{dN(r),dN(s)\}$

$\sim p_N \delta\{u\}h + q_{NN}(u)h^2$

as $\mathrm{cov}\{dN(r),dN(s)\} = [p_N \, \delta(r-s) + q_{NN}(r-s)]dr \, ds$,

rate, p_N, covariance density, $q_{NN}(\cdot)$, Dirac delta, $\delta(\cdot)$

Parametric models.

Bernoulli ARCH. Cox (1970)

$\text{Prob}\{Z_t = 1|H_t\} = \pi_t$

$\text{logit } \pi_t = \Sigma\, \alpha_i Z_{t-i}$

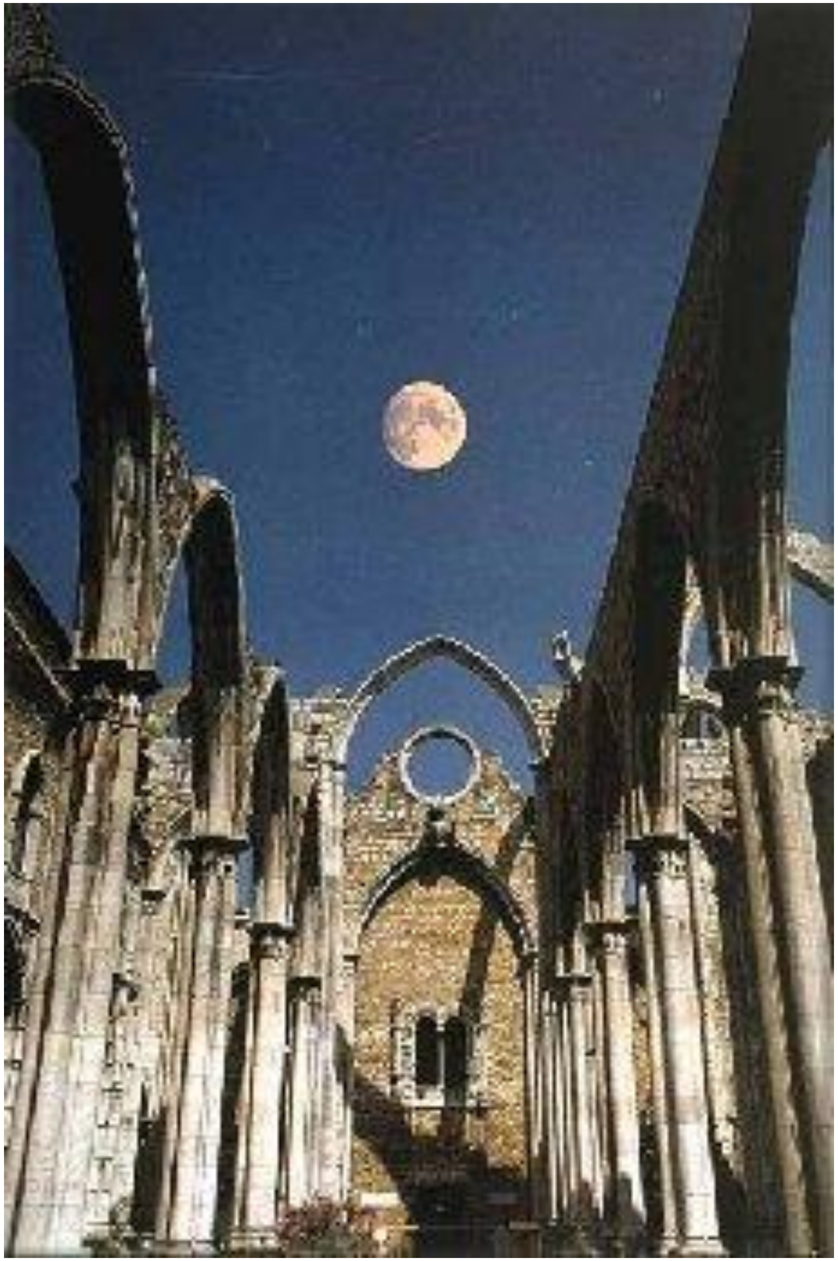
H_t history before t

Fitting, assessment, prediction, … via glm()

Bernoulli GARCH.

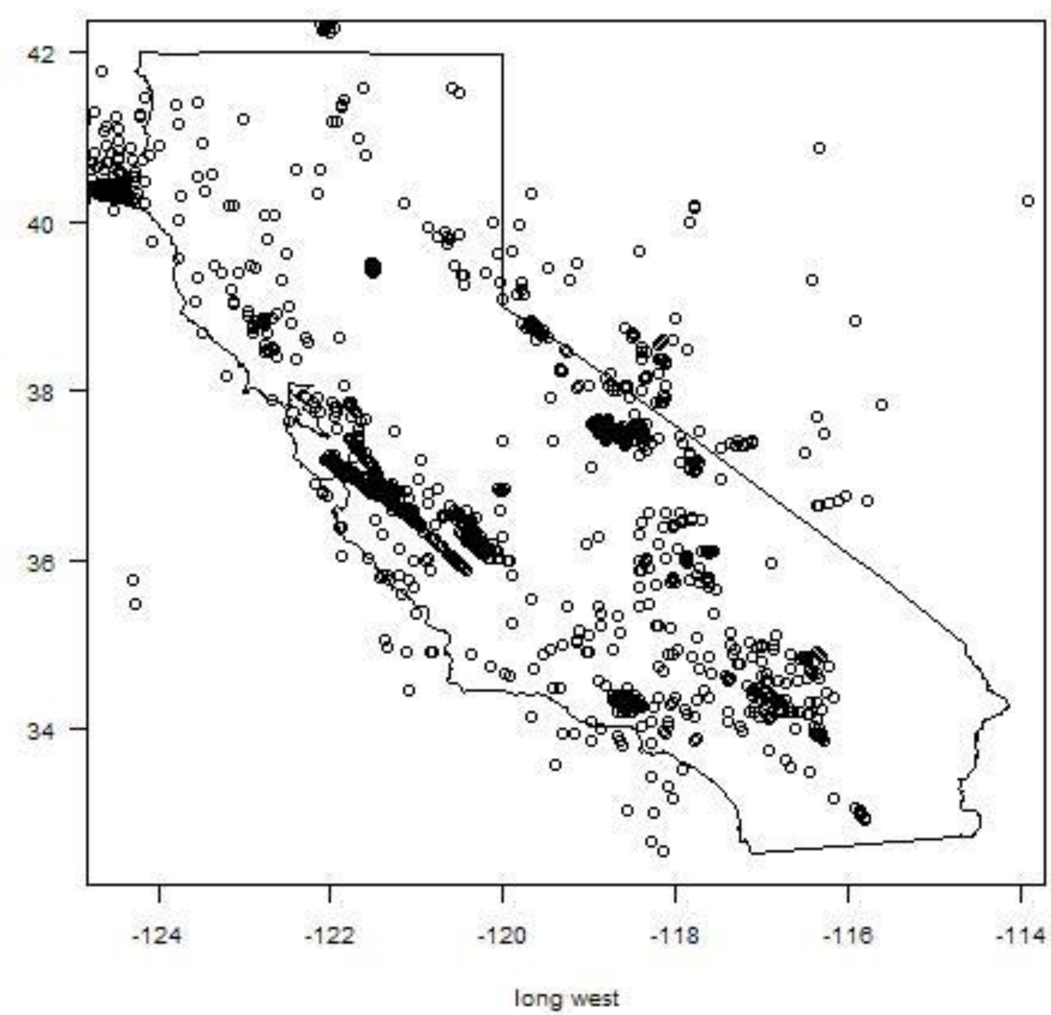
$\text{logit } \pi_t = \Sigma\, \alpha_i Z_{t-i} + \Sigma\, \beta_j \text{ logit } \pi_{t-j}$

Volatility π_t or $\pi_t(1 - \pi_t)$?



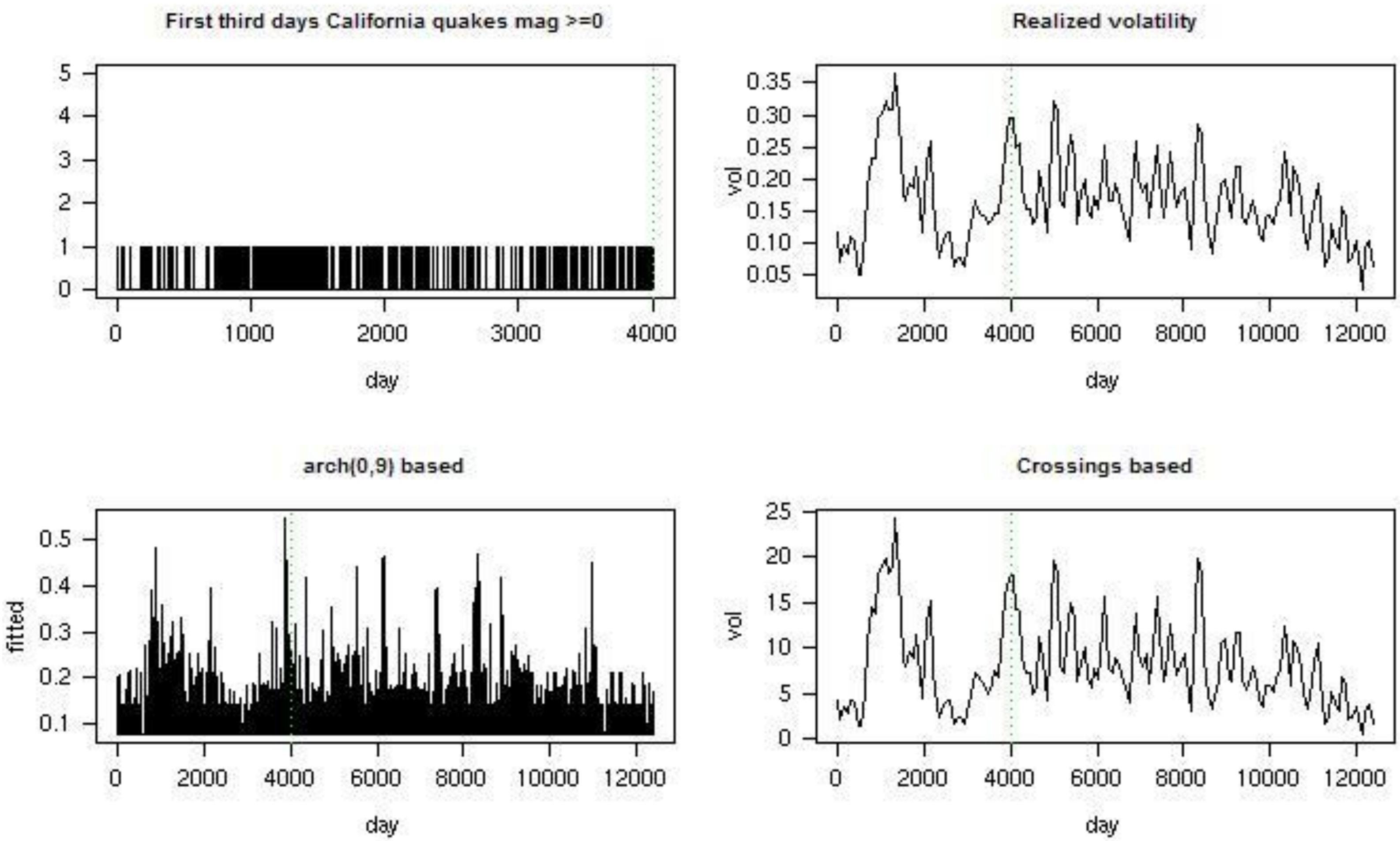
Convento do Carmo

“California” earthquakes magnitude $\geq$ 4, 1969-2003



N=1805

Results of 0-1 analysis



P.p. analysis.

Rate as estimate of volatility

Consider var{dN(t)}

$$\text{var}\{N(t)-N(t-h)\} \approx p_N h + q_{NN}(0)h^2$$

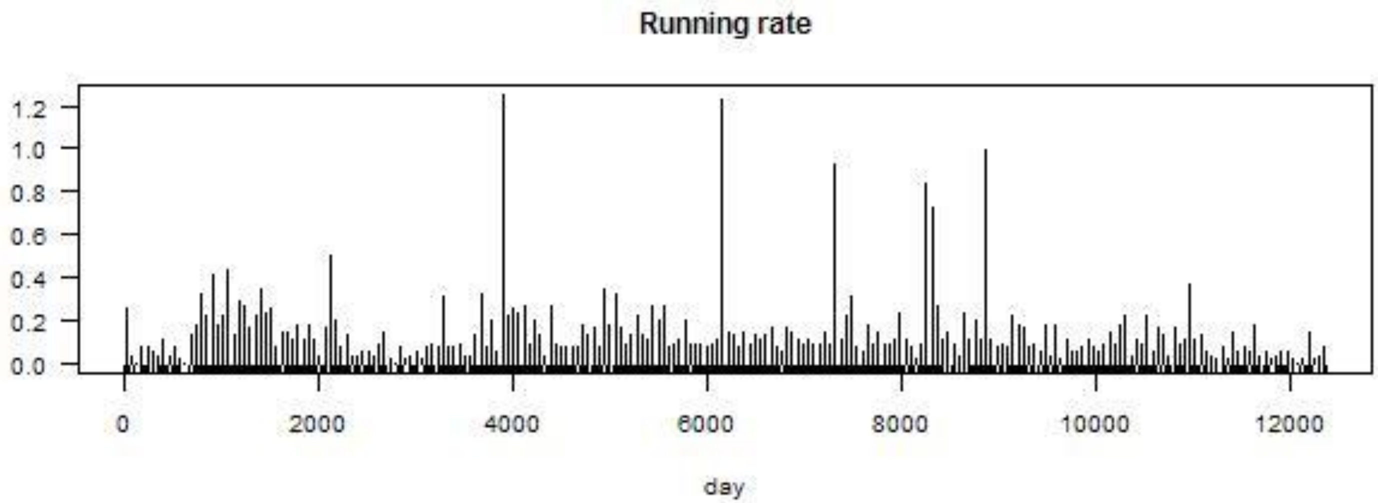
Estimate of rate at time t.

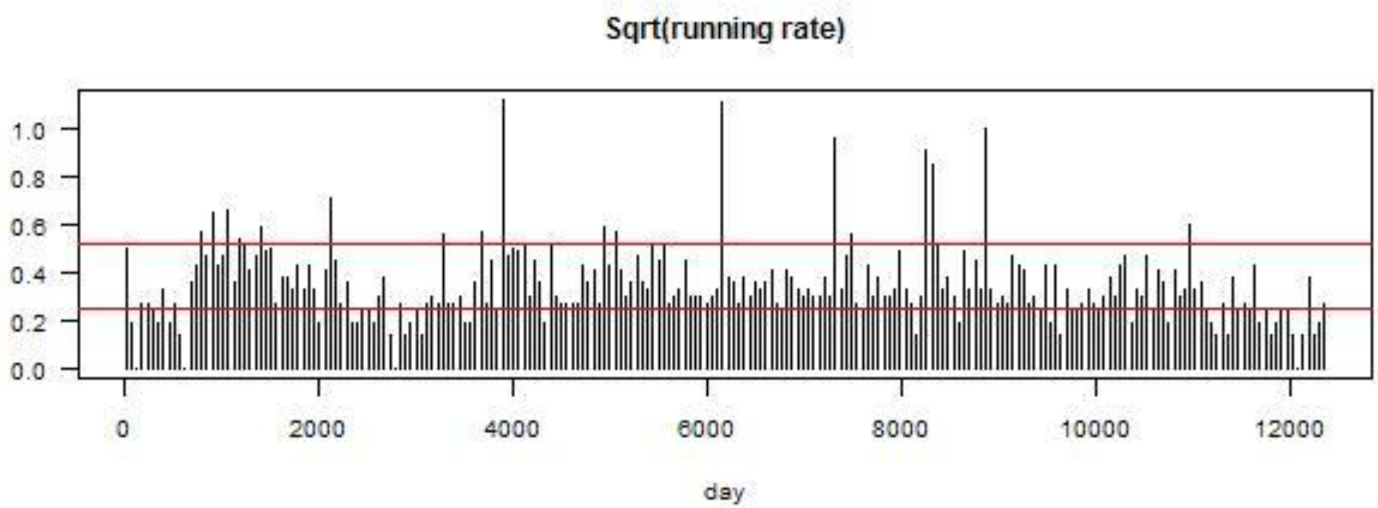
$$\int k(t-u)dN(u) \,/\, \int k(t-u)du$$

k(.) kernel

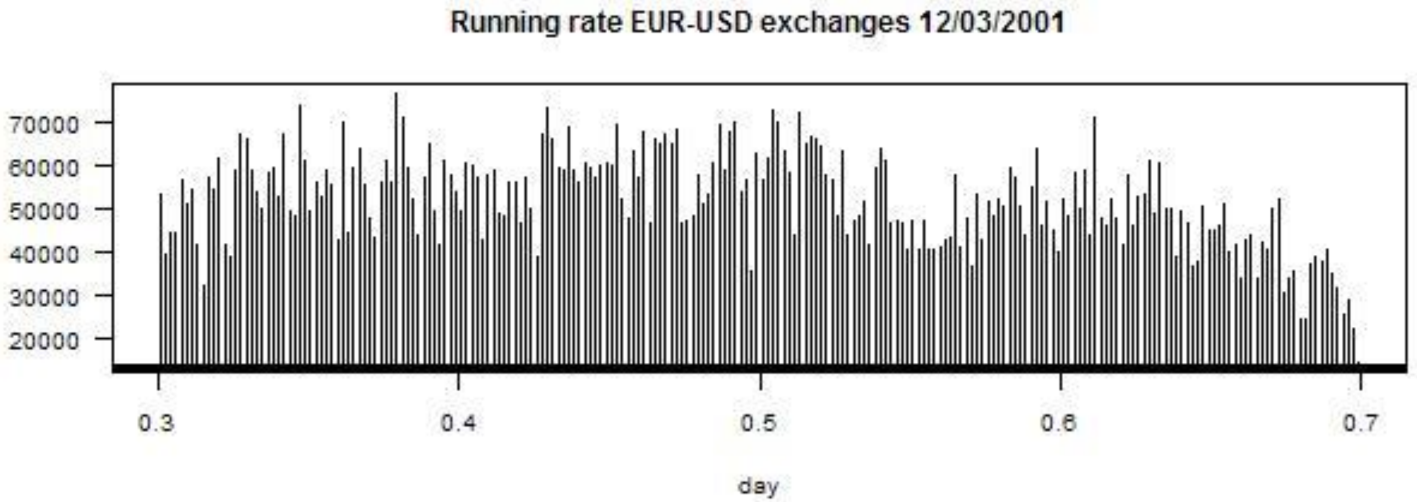
Variance, stationary case, k(.) narrow

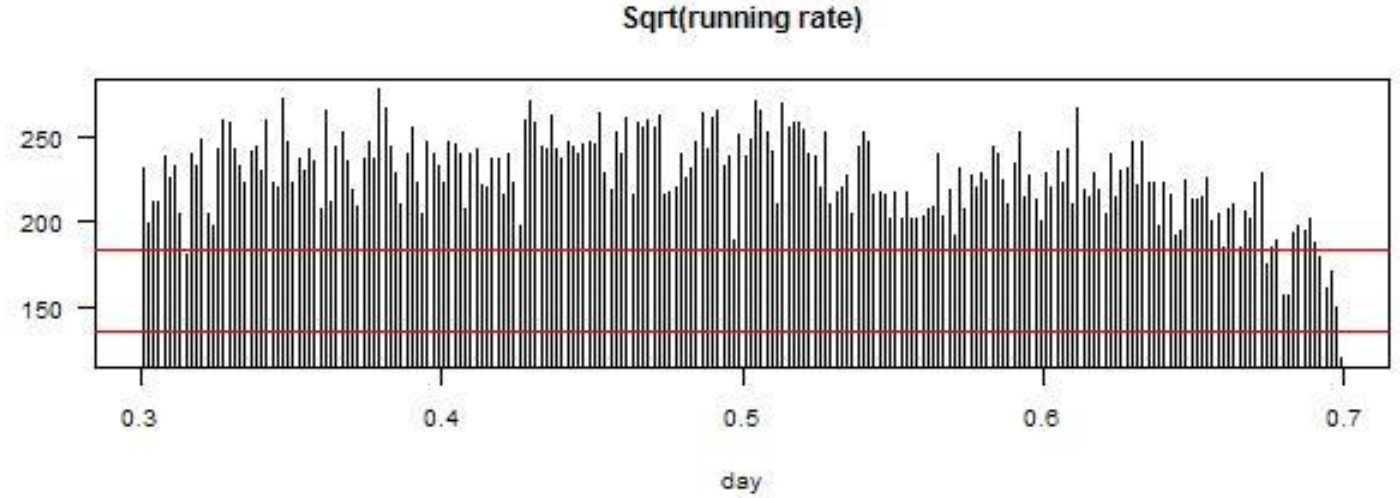
$$p_N \int k(t-u)^2 du \,/\, [\int k(t-u)du]^2 + q_{NN}(0)$$


Running rate
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
2000
4000
6000
8000
10000
12000
day


Sqrt(running rate)
1.0
0.8
0.6
0.4
0.2
0.0
0
2000
4000
6000
8000
10000
12000
day

Example. Euro-USA exchange rate



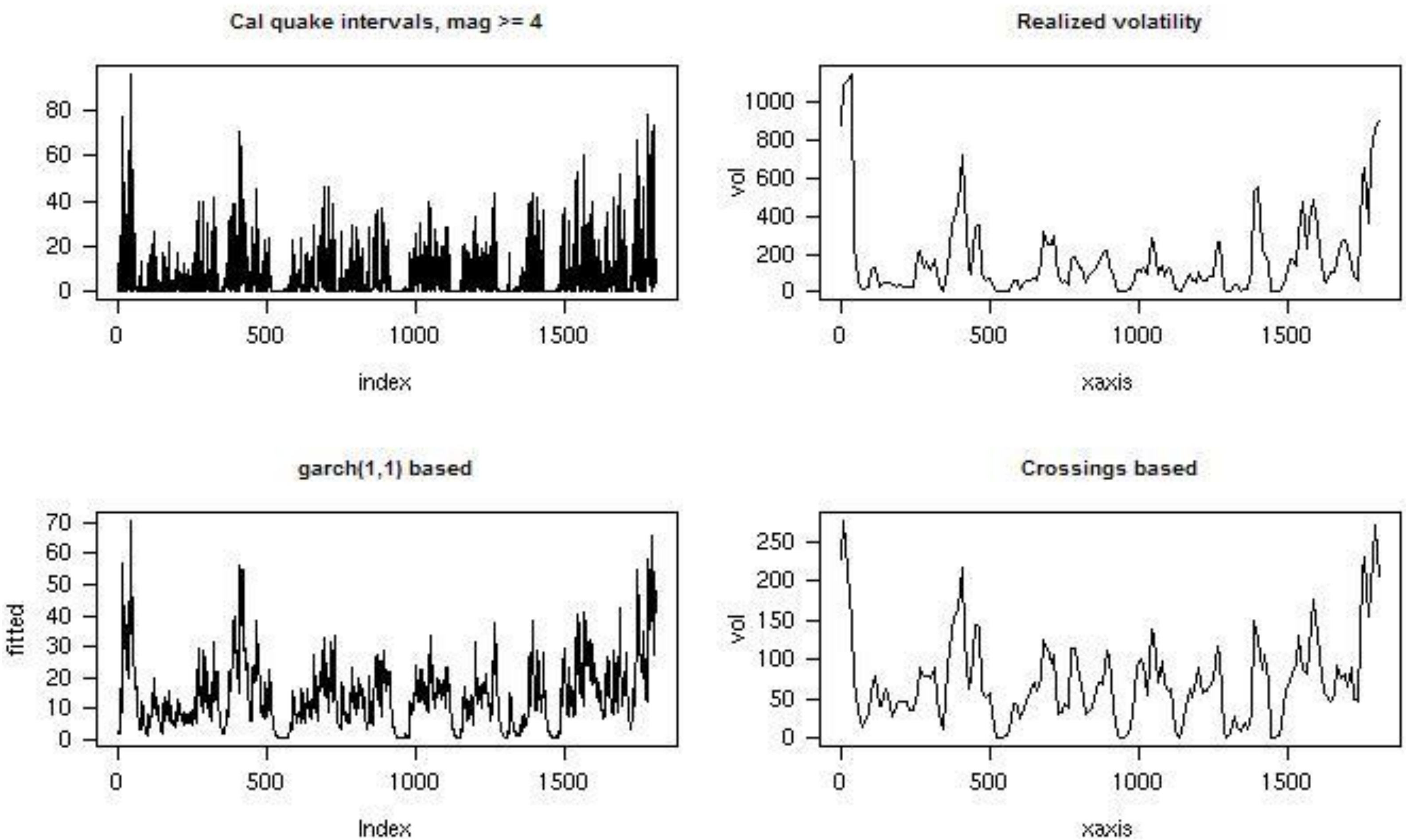


Interval analysis.

$$X_j = \tau_j - \tau_{j-1}$$

Also stationary

California earthquakes



Risk analysis. *Time series case*.

Assets Y_t and probability p

VaR is the p-th quantile

$$\text{Prob}\{Y_{t+1} - Y_t \leq \text{VaR}\} = p$$

left tail

If approxmate distribution of $Y_{t+1} - Y_t$ by

Normal$(0,\sigma_t)$

volatility, σ_t, appears

Sometimes predictive model is built and fit to estimate VaR

Point process case.

Pulses arriving close together can damage

Number of oscillations to break object (Ang & Tang)

Suppose all points have the same value (mark), e.g. spike train.

Consider VaR of

$$\text{Prob}\{N(t+u) - N(t) > \text{VaR}\} = 1\text{-}p$$

Righthand tail

Examples.

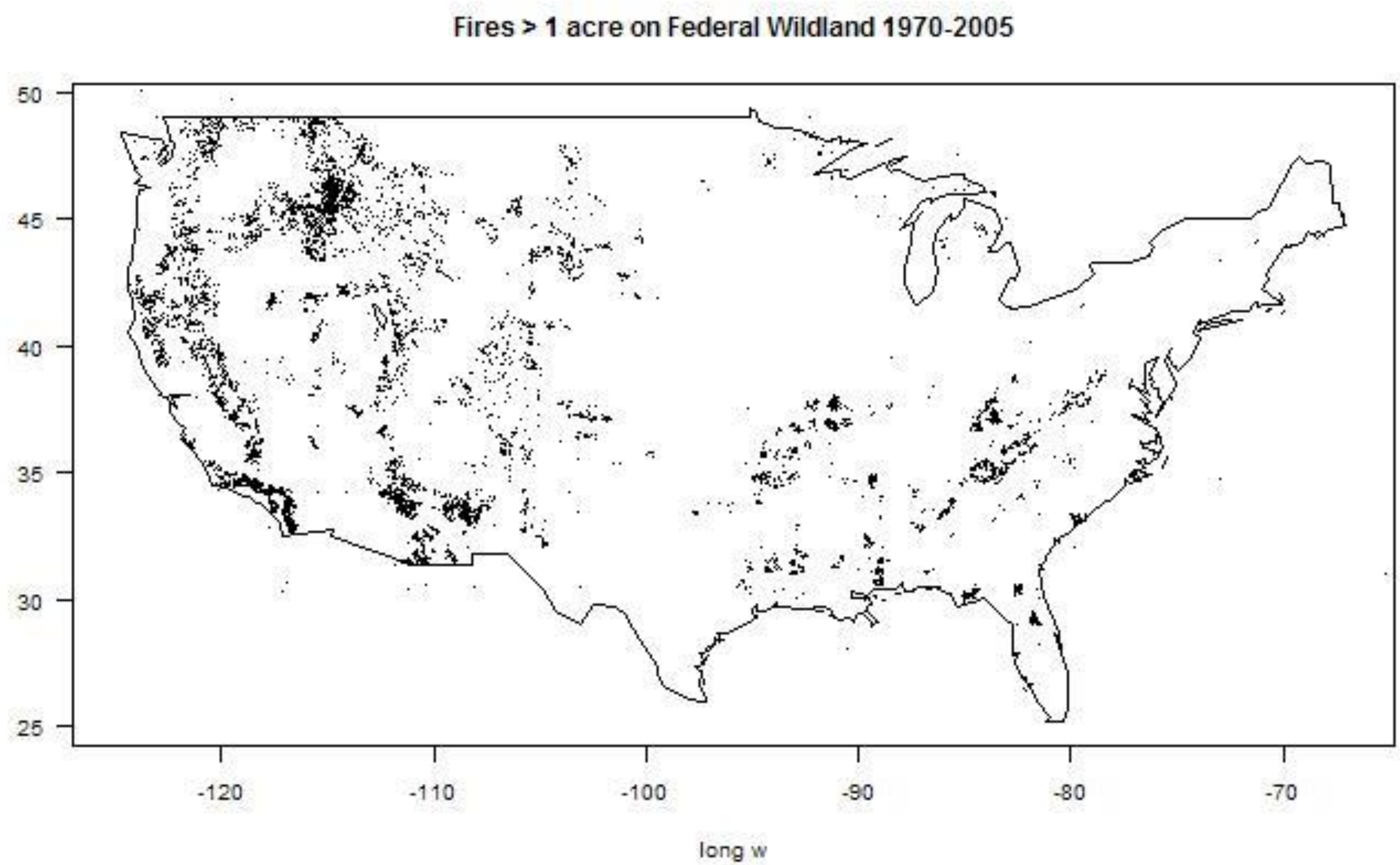
S&P500: p=.05 method of moments quantile

VaR = $.0815

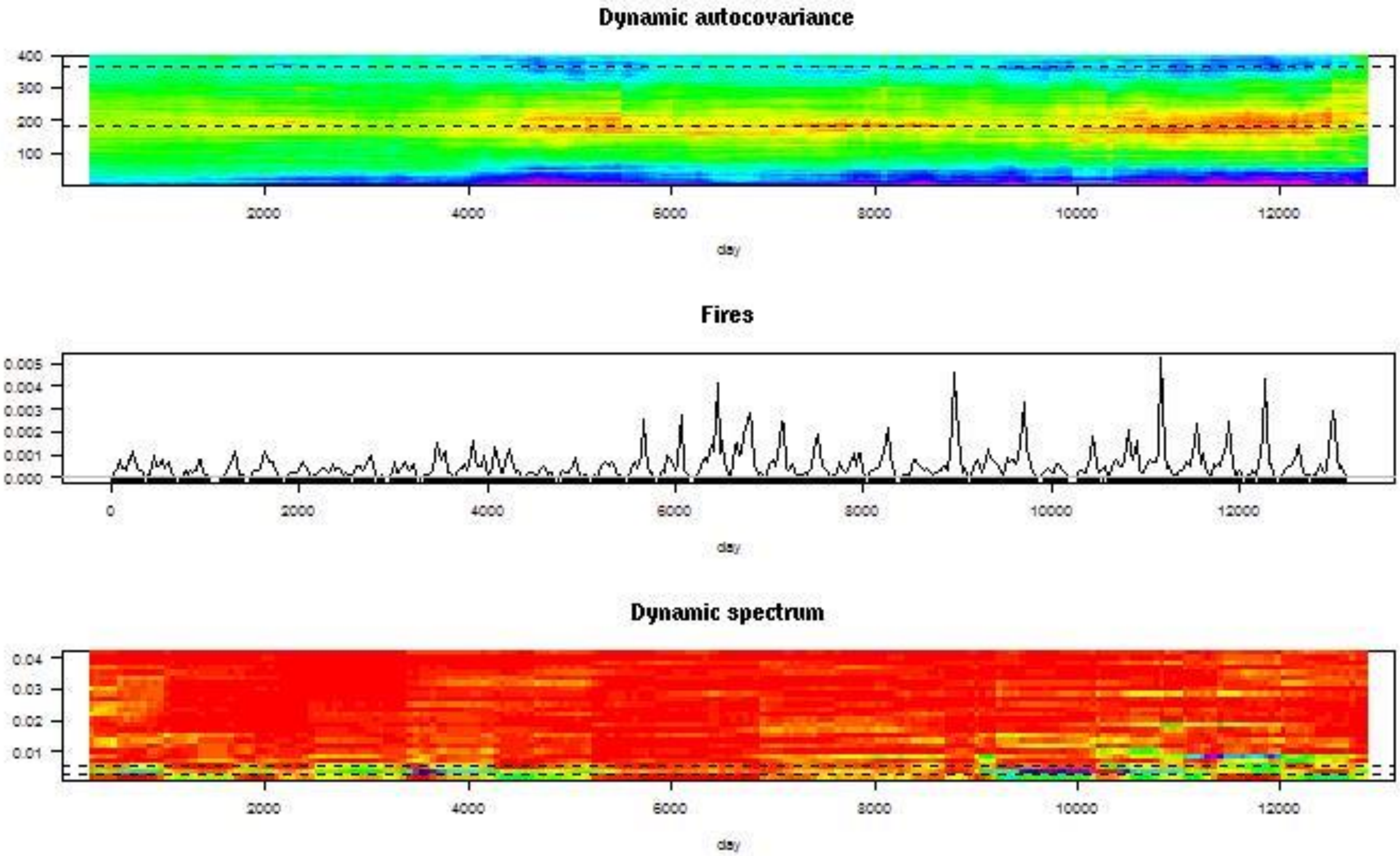
CA earthquakes: u = 7 days, p=.95 mom quantile

VaR = 28 events

Case with seasonality – US Forest Fires 1970 - 2005

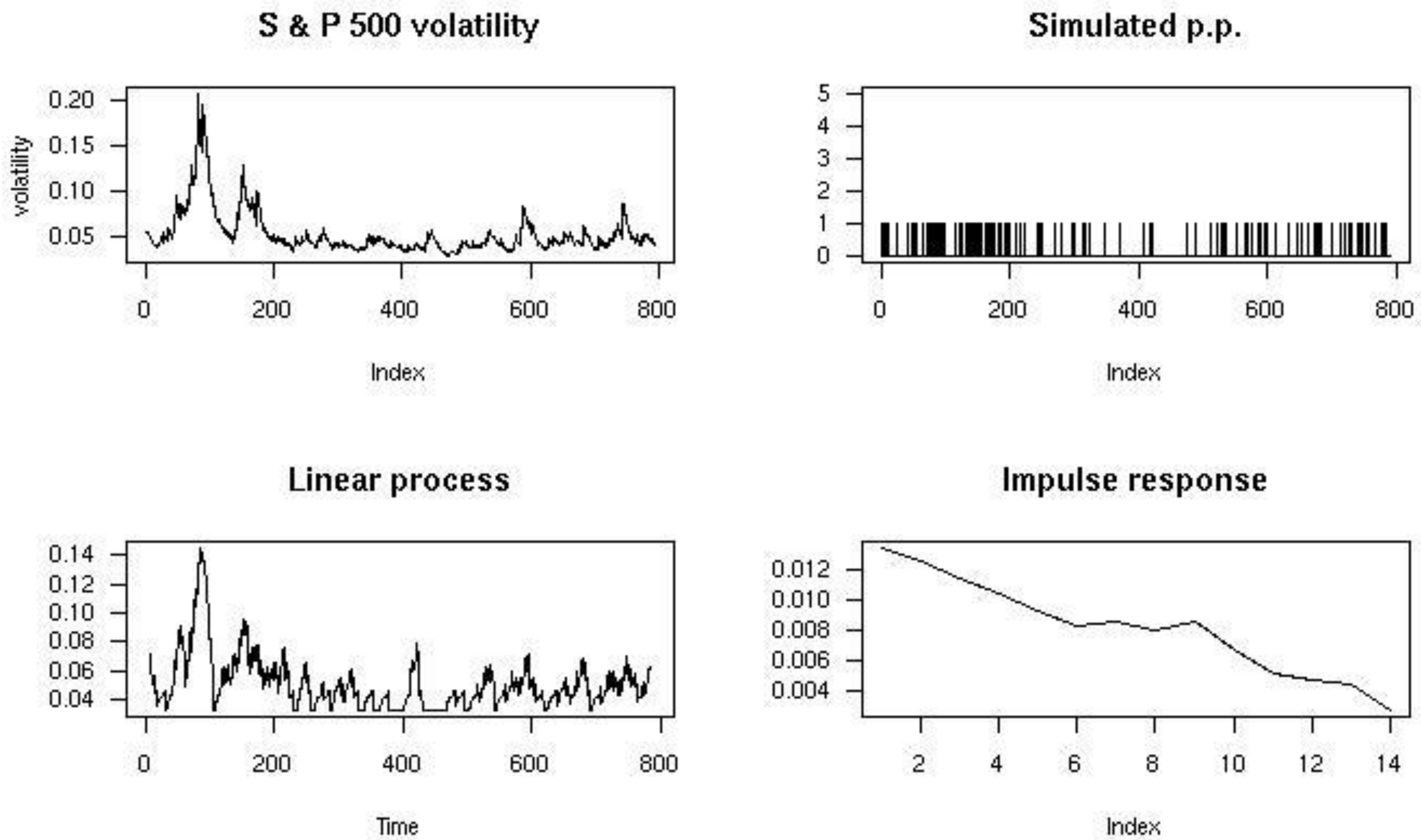


n=8481



Dashed line ~ seasonal

Example. Volatility, simulate Poisson, recover volatility



Conclusion.

Returning to the question, “Why bother with extension?

a) Perhaps will learn more about time series case

b) Pps are an interesting data type

c) Pps are building blocks

d) Volatility often considered risk measure for time series.”

The volatility can be the basic phenomenon

Another question**.**

“Is there a useful extension of the concept of volatility to point processes?”

The running rate

Gets at local behavior (prediction)

Some references.

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Tsay, R. S. (2002). Analysis of Financial Time Series. Wiley