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Kalman recursions.

Proof (Text's)

$$\underset{\sim}{I}_t = \underset{\sim}{Y}_t - \underset{\sim}{P}_{t-1} \underset{\sim}{Y}_t \quad \text{orthogonal}$$

$$= \underset{\sim}{Y}_t - \underset{\sim}{G}_t \hat{\underset{\sim}{X}}_t$$

$$= \underset{\sim}{G}_t (\underset{\sim}{X}_t - \hat{\underset{\sim}{X}}_t) + \underset{\sim}{W}_t$$

$$P(\cdot \mid \ldots, \underset{\sim}{Y}_t)$$

$$= P(\cdot \mid \ldots, \underset{\sim}{Y}_{t-1}) + P(\cdot \mid \underset{\sim}{I}_t)$$

$$\hat{\underset{\sim}{X}}_{t+1} = P_t(\underset{\sim}{X}_{t+1}) \qquad \text{def}^{\underline{n}}$$

$$= P_{t-1}(\underset{\sim}{X}_{t+1}) + P(\underset{\sim}{X}_{t+1} \mid \underset{\sim}{I}_t)$$

$$\underset{\sim}{\Delta}_t = E\,\underset{\sim}{I}_t \underset{\sim}{I}_t' = \underset{\sim}{G}_t \underset{\sim}{\Omega}_t \underset{\sim}{G}_t' + \underset{\sim}{R}_t$$

$$\underset{\sim}{\Theta}_t = E\,\underset{\sim}{X}_{t+1} \underset{\sim}{I}_t' = \underset{\sim}{F}_t \underset{\sim}{\Omega}_t \underset{\sim}{G}_t'$$

$$P(\underset{\sim}{X}_{t+1} \mid \underset{\sim}{I}_t) = \underset{\sim}{\Theta}_t \underset{\sim}{\Delta}_t^{-1} \underset{\sim}{I}_t$$

$$P_{t-1}(\underset{\sim}{X}_{t+1}) = P_{t-1}(\underset{\sim}{F}_t \underset{\sim}{X}_t + \underset{\sim}{V}_t)$$

$$= \underset{\sim}{F}_t \hat{X}_t$$

giving (8.4.1)

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For (8.4.2)

$$\underset{\sim}{\Omega}_{t+1} = E\left[\left(\underset{\sim}{X}_{t+1} - \hat{\underset{\sim}{X}}_{t+1}\right)\left(\underset{\sim}{X}_{t+1} - \hat{\underset{\sim}{X}}_{t+1}\right)'\right]$$

$$= E\left(\underset{\sim}{X}_{t+1}\underset{\sim}{X}_{t+1}'\right) - E\left(\hat{X}_{t+1}\hat{X}_{t+1}'\right) \quad (*)$$

$$\left\{\begin{array}{l} E(U) = \underset{A}{E}\left(E(U|A)\right) \\ \mathrm{Var}(U) = \underset{A}{E}\,\mathrm{Var}(U|A) + \underset{A}{\mathrm{Var}}\left(E(U|A)\right) \end{array}\right\}$$

$$(*) = \underset{\sim}{F}_t E\left(\underset{\sim}{X}_t\underset{\sim}{X}_t'\right)\underset{\sim}{F}_t' + \underset{\sim}{Q}_t$$

$$- \underset{\sim}{F}_t E\left(\hat{\underset{\sim}{X}}_t\hat{\underset{\sim}{X}}_t'\right)\underset{\sim}{F}_t' - \underset{\sim}{\Theta}_t \underset{\sim}{\Delta}_t^{-1} \underset{\sim}{\Theta}_t'$$

(8.4.2)