Some interpretations of spectral concepts.

Ordinary statistics concepts, r.v. $Y$

variance, $\text{var } Y = \sigma_y^2$

linear model, $Y = \beta X + \epsilon$

Suppose $(X, \epsilon)$ r.v with $X \perp \epsilon$ means 0

regression coefficient, $E(Y|X) = \beta X$

$\beta$ may be positive or negative

$\beta = \frac{\sigma_{yx}}{\sigma_{xx}}$

min $\text{MSE}$, $\min \{E(Y - \beta X)^2\}$

$\beta = \frac{\sigma_{yx}}{\sigma_{xx}}$
minimum achieved

\[(1 - \rho^2) \sigma_{yy} = \sigma_{ee} \]

correlation coefficient

\[\rho = \frac{\sigma_{yx}}{\sqrt{\sigma_{yy} \sigma_{xx}}}\]

\[|\rho| \leq 1\]

coefficient of determination

\[\rho^2\]

measure of strength of linear association
Spectral concepts. Stationary t.o. \( Y(t) \)

Cramér representation. By random

\[
Y(t) = \int_{-\pi}^{\pi} e^{it\lambda} dZ_{\lambda}(t)
\]

Spectrum

\[
\text{cov}\{dZ_{\lambda}(t), dZ_{\mu}(\omega)\} = S(\lambda - \mu) f_{\lambda\mu}(\omega) d\lambda
\]

\(-\pi < \lambda, \mu < \pi\)

\[
f_{\lambda\mu}(\lambda) \propto \var\{dZ_{\lambda}(t)\}^2
\]

\[
E|dZ_{\lambda}(t)|^2
\]

Linear time invariant model.

\[
Y(t) = \sum_{n} a(n) X(t-n) + \epsilon(t)
\]

\((X(t), \epsilon(t))\) stationary, mean 0, \( X(l) \in \mathbb{R} \)

\[
E\{Y(t) | X\} = \sum_{n} a(n) X(t-n)
\]
Transfer function:
\[ dZ_y(t) = A(t) dZ_x(t) + dZ_e(t) \]

\[ E[dZ_y(t) | X^2] = A(t) dZ_x(t) \]

\( A(t) \) is a regression coefficient.

One is carrying out regressions separately for each frequency.

\( A(t) \) is complex-valued.

**gain**: \( |A(t)| \)  
**phase**: \( \angle A(t) \) in \( [-\pi, \pi] \)

e.g. \( Y(t) = a * X(t-5) \), \( a > 0 \)

\[ A(t) = a e^{-i\omega t} \]

\( \angle A(t) = -\omega t \pmod{2\pi} \)

\[ A(t) = \frac{f_{yx}(t)}{f_{xx}(t)} \]

\[ f_{yx}(t) \propto \text{cor} \{ dZ_y(t), dZ_x(t) \} \]

\[ E[YX] \]
\[
\min_{\mathbf{\alpha}} \text{MSE} = \min_{\mathbf{\alpha}} E \left( Y(\tau) - \sum_{n} a(n) X(\tau-n) \right)^2
\]

\[
A(\alpha) = \frac{f_{yx}(\alpha)}{f_{xx}(\alpha)}
\]

Minimum achieved
\[
\int (1 - |R_{yx}(\alpha)|^2) f_{yy}(\eta) \, d\eta
\]

\[
\text{coherence} \quad R_{yx}(\alpha) = \frac{f_{yx}(\alpha)}{\sqrt{f_{xx}(\alpha)f_{yy}(\alpha)}}
\]

\[
\text{coherence} \quad |R_{yx}(\alpha)|^2 \leq 1
\]

\[
\xi_{yx}(\alpha) = \sqrt{1 - |R_{yx}(\alpha)|^2} f_{yy}(\eta)
\]

Coherence is a measure of the strength of linear time-invariant association.