 30 March 04

Some interpretations of spectral concepts

Ordinary statistics concepts, r.v. Y

variance, $\mathrm{var}\, Y = \sigma_{YY}$

linear model, $Y = \beta X + \epsilon$

suppose (X, ϵ) r.v with $X \perp\!\!\!\perp \epsilon$

means 0

regression coefficient,

$E(Y|X) = \beta X$

β may be positive or negative

$\beta = \sigma_{YX} / \sigma_{XX}$

min MSE, $\min_{\beta} E(Y - \beta X)^2$

$\beta = \sigma_{YX} / \sigma_{XX}$

minimum achieved

$$(1-\rho^2)\,\sigma_{YY} = \sigma_{\epsilon\epsilon}$$

<u>correlation coefficient</u>

$$\rho = \sigma_{YX} / \sqrt{\sigma_{YY}\,\sigma_{XX}}$$

$$|\rho| \le 1$$

<u>coefficient of determination</u>

$$\rho^2$$

measure of strength of linear association

30 March 04

Spectral concepts. stationary t.s. $Y(t)$

Cramér representation. Z_Y random

$$Y(t) = \int_{-\pi}^{\pi} e^{it\lambda}\, dZ_Y(\lambda)$$

spectrum

$$\operatorname{cov}\{dZ_Y(\lambda), dZ_Y(\mu)\} = \delta(\lambda-\mu)\, f_{YY}(\lambda)\, d\lambda$$

$$-\pi < \lambda, \mu < \pi$$

$$f_{YY}(\lambda) \quad \propto \quad \operatorname{var}\{dZ_Y(\lambda)\}$$

$$E\,|dZ_Y(\lambda)|^2$$

linear time invariant model.

$$Y(t) = \sum_u a(u)\, X(t-u) + \epsilon(t)$$

$(X(t), \epsilon(t))$ stationary, means 0, $X \perp\!\!\!\perp \epsilon$

$$E\{Y(t) \mid X\} = \sum_u a(u)\, X(t-u)$$

30 March 04

<u>transfer function</u>:

$$dZ_Y(\lambda) = A(\lambda)\, dZ_X(\lambda) + dZ_\epsilon(\lambda).$$

$$E\{dZ_Y(\lambda) \mid X\} = A(\lambda)\, dZ_X(\lambda)$$

$A(\lambda)$ is a regression coefficient

One is carrying out regressions separately for each frequency

$A(\lambda)$ is complex-valued

gain: $|A(\lambda)|$ phase: $\arg A(\lambda)$ in $(-\pi, \pi]$

e.g. $Y(t) = a X(t-\sigma)$, $a > 0$

$$A(\lambda) = a e^{-i\lambda\sigma}$$

$$\arg A(\lambda) = -\lambda\sigma \pmod{2\pi}$$

$$A(\lambda) = f_{YX}(\lambda) / f_{XX}(\lambda)$$

$$f_{YX}(\lambda) \propto \operatorname{cov}\{dZ_Y(\lambda), dZ_X(\lambda)\}$$

$$E\{Y\bar{X}\}$$

min MSE.

$$\min_a E\left(Y(t) - \sum_u a(u) X(t-u)\right)^2$$

$$A(\lambda) = f_{YX}(\lambda) / f_{XX}(\lambda)$$

minimum achieved

$$\int (1 - |R_{YX}(\lambda)|^2) f_{YY}(\lambda) \, d\lambda$$

coherency

$$R_{YX}(\lambda) = f_{YX}(\lambda) / \sqrt{f_{XX}(\lambda) f_{YY}(\lambda)}$$

coherence $|R_{YX}(\lambda)|^2 \leq 1$

$$f_{\epsilon\epsilon}(\lambda) = [1 - |R_{YX}(\lambda)|^2] f_{YY}(\lambda)$$

coherence is a measure of the strength of linear time-invariant association