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<u>State eq$^{\underline{n}}$</u>. $\underset{\sim}{Y}_t = \underset{\sim}{G}_t \underset{\sim}{X}_t + \underset{\sim}{W}_t$

<u>Obs$^{\underline{n}}$ eq$^{\underline{n}}$</u>. $\underset{\sim}{X}_{t+1} = \underset{\sim}{F}_t \underset{\sim}{X}_t + \underset{\sim}{V}_t$

<u>Innovations</u>. $\underset{\sim}{I}_t = \underset{\sim}{Y}_t - P_{t-1} \underset{\sim}{Y}_t$

$\{\underset{\sim}{I}_t\}$ is orthogonal

independent if $\{\underset{\sim}{W}_t, \underset{\sim}{V}_t\}$ Gaussian

$E(\underset{\sim}{I}_t \underset{\sim}{I}_t') = \underset{\sim}{\Delta}_t$

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Eg. AR(1).

$$Y_t = \varphi Y_{t-1} + \sigma Z_t$$

$\{Z_t\}$ independent, $p(\cdot)$

$$f(Y_t | Y_{t-1}, \dots, Y_0) = f(Y_t | Y_{t-1})$$

$$= \frac{1}{\sigma} p\left(\frac{Y_t - \varphi Y_{t-1}}{\sigma}\right)$$

$$L(\theta; Y_1, \dots, Y_n | Y_0)$$

$$= \frac{1}{\sigma} p\left(\frac{Y_1 - \varphi Y_0}{\sigma}\right) \frac{1}{\sigma} p\left(\frac{Y_2 - \varphi Y_1}{\sigma}\right) \cdots \frac{1}{\sigma} p\left(\frac{Y_n - \varphi Y_{n-1}}{\sigma}\right)$$

Gaussian case

$$-2 \log L = n \log \sigma^2 + \sum_t (Y_t - \varphi Y_{t-1})^2 / \sigma^2$$

$$I_t = \sigma Z_t$$

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$$\frac{\partial}{\partial\varphi}: \quad \hat{\varphi} = \sum_t Y_t Y_{t-1} \Big/ \sum_t Y_{t-1}^2$$

$$\frac{\partial}{\partial\sigma^2}: \quad \hat{\sigma}^2 = \sum_t (Y_t - \hat{\varphi} Y_{t-1})^2 / n$$

cf. regression
least squares
s.e.

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(8.6.1), (8.1.2) state space model

Set $\underset{\sim}{y}_t^* = \underset{\sim}{y}_t \quad t \in \{i_1, \ldots, i_r\}$

$= 0$ otherwise

Data $\{\underset{\sim}{y}_t, t = i_1, \ldots, i_r\}$

$L_1(\theta; \underset{\sim}{y}_{i_1}, \ldots, \underset{\sim}{y}_{i_r})$

$= (2\pi)^{(n-r)w/2} L_2(\theta; \underset{\sim}{y}_1^*, \ldots, \underset{\sim}{y}_r^*)$

Filled in missing with $\underset{\sim}{0}$

Apply Kalman to (8.6.1), (8.1.2)

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AR(1) given y_1, y_3, y_4, y_5

$$L = \sigma^{-4} (2\pi)^{-2} \left[(1-\varphi^2)/(1+\varphi^2) \right]^{\frac{1}{2}}$$

$$\exp\left\{ -\frac{1}{2\sigma^2} \left[y_1^2 (1-\varphi^2) + \frac{(y_3 - \varphi^2 y_1)^2}{1+\varphi^2} \right.\right.$$

$$\left.\left. + (y_4 - \varphi y_3)^2 + (y_5 - \varphi y_4)^2 \right] \right\}$$

$$Y_t = \varphi Y_{t-1} + W_t$$

$$= W_t + \varphi W_{t-1} + \varphi^2 W_{t-2} + \cdots$$

$$EY_t = 0, \quad \mathrm{Var}\, Y_t = (1 + \varphi^2 + \varphi^4 + \ldots)\sigma^2$$

$$= \sigma^2 / (1-\varphi^2)$$

$$p_{Y_1}(y) = \frac{1}{\sqrt{2\pi}} \sqrt{\frac{1-\varphi^2}{\sigma^2}} \exp\left\{ -\frac{(1-\varphi^2)}{2\sigma^2} y^2 \right\}$$

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$$Y_3 = \varphi Y_2 + W_3$$

$$= \varphi^2 Y_1 + W_3 + \varphi W_2$$

$$Y_3 - \varphi^2 Y_1 = W_3 + \varphi W_2 \perp\!\!\!\perp Y_1 = \varphi Y_0 + W_1$$

$$\sim N(0, (1+\varphi^2)\sigma^2)$$

$$Y_4 - \varphi Y_3 = W_4 \perp\!\!\!\perp Y_1, Y_3$$

$$\sim N(0, \sigma^2)$$

$$Y_5 - \varphi Y_4 = W_5 \perp\!\!\!\perp Y_1, Y_3, Y_4$$