Statistics 248 - Homework 6 (D. R. Brillinger)

Consider the state space model

$$\mathbf{S}(t) = \mathbf{F}\mathbf{S}(t-1) + \Psi\varepsilon(t)$$
$$Y(t) = [1\ 0_{r-1}]\mathbf{S}(t)$$

with $\mathbf{S}(.)$ $r$-vector valued and $\varepsilon(.)$ real-valued white noise. $0_{r-1}$ is the row vector of $r-1$ $0's$.

1. Show that the following particular cases correspond to the indicated models:

**MA(1).**

$$\mathbf{F} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \Psi = \begin{bmatrix} b(0) \\ b(1) \end{bmatrix}$$

**AR(1).**

$$\mathbf{F} = [-a(1)] \qquad \Psi = [1]$$

**ARMA(1,1).**

$$\mathbf{F} = \begin{bmatrix} -a(1) & 1 \\ 0 & 0 \end{bmatrix} \qquad \Psi = \begin{bmatrix} b(0) \\ b(1) \end{bmatrix}$$

2. Make an attempt to write the general ARMA(p,q) in state space form. (Notes: 1. $r$ will be $\max\{p, q+1\}$. 2. Consider AR(p) and MA(q) to begin, perhaps.)