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Statistics 248 - Homework 6

a) Show that if $\{X_t\}$ and $\{Y_t\}$ are independent (second-order) stationary processes with power spectral density functions $f_x(\lambda)$ and $f_y(\lambda)$, then $\{V_t\} = \{X_t + Y_t\}$ is also (second-order) stationary with power spectral density function $f_v(\lambda) = f_x(\lambda) + f_y(\lambda)$.

b) If

$$V_t = X_t + Y_t$$

where

$$X_t = \alpha X_{t-1} + W_t \qquad -1 < \alpha < +1$$

and $\{Y_t\}$, $\{W_t\}$ are independent purely random processes with zero mean and common variance σ^2, show that the power spectral density function of $\{V_t\}$ is given by

$$f_v(\lambda) = \sigma^2(2 - 2\alpha\cos\lambda + \alpha^2)/2\pi(1 - 2\alpha\cos\lambda + \alpha^2)$$

$0 < \lambda < \pi$.