Statistics 248 - Homework 3 (D. R. Brillinger)

Let **A** be an linear time invariant operator on functions of time, $Y(t)$, $-\infty < t < \infty$. Let $A(\lambda)$ denote its transfer function. Suppose that **A** leaves unchanged all polynomials

$$Y(t) = \sum_{j=0}^{n} \alpha_j t^j$$

i.e. $\mathbf{A}[Y] = Y$.

Show that this happens if and only if the transfer function is such that

$$A(0) = 1$$

and

$$A^{(j)}(0) = 0$$

for $0<j\leq n$ and the superscript j referring to taking the derivative j times.