Homework 1.

1. (Problem 2G.2) Let $\Phi_s(\omega)$ be the (power) spectrum of a scalar signal defined by

$$\Phi_s(\omega) \;=\; \sum_{\tau=-\infty}^{\infty} R_s(\tau) e^{-i\tau\omega}$$

Take $\{s(t)\}$ to be a stationary stochastic process with mean 0 and $cov\{s(t), s(t-\tau)\} \;=\; R_s(\tau)$.
Show that

$$i. \quad \Phi_s(\omega) \; is \; real.$$

$$ii. \quad \Phi(\omega) \;\geq\; 0, \quad for \; all \; \omega$$

$$iii. \quad \Phi_s(-\omega) \;=\; \Phi_s(\omega)$$

Hint, with

$$S_N(\omega) \;=\; \frac{1}{\sqrt{N}} \sum_{t=1}^{N} s(t) e^{-i\omega t}$$

consider the limit as $N \;\rightarrow\; \infty$ of

$$E\{|S_N(\omega)|^2\}$$

2. (Problem 3D.2) Establish

$$\frac{1}{N} \sum_{k=1}^{N} e^{2\pi i r k/N} \;=\; 1 \; if \; r \;=\; 0$$

$$= 0 \; if \; r \;=\; 1, ..., N-1$$