20 Feb. 2003

# Chapt. 8. State-space Models

$$\{\underset{\sim}{W}_t\}^{w\times 1} \sim WN(\underset{\sim}{0}, \{\underset{\sim}{R}_t\})$$

$\underset{\sim}{Y}_t^{w\times 1}$ observation vector

$\underset{\sim}{X}_t^{v\times 1}$ state vector

## State equation.

$$\underset{\sim}{X}_{t+1} = \underset{\sim}{F}_t \underset{\sim}{X}_t + \underset{\sim}{V}_t$$

$$\{\underset{\sim}{V}_t\} \sim WN(\underset{\sim}{0}, \{\underset{\sim}{Q}_t\})$$

$$E(\underset{\sim}{W}_t \underset{\sim}{V}_s') = \underset{\sim}{0}$$

# State space representation

**Observation equation**

$$\underset{\sim}{Y}_t = \overset{w \times r}{\underset{\sim}{G}_t} \, \underset{\sim}{X}_t + \underset{\sim}{W}_t$$

---

$$\underset{\sim}{X}_{t+1} = \underset{\sim}{F}_t \, \underset{\sim}{X}_t + \underset{\sim}{V}_t$$

$\underset{\sim}{X}_t$ - all the information

20 Feb 2003

If independence,

$\{\underset{\sim}{X}_t\}$ Markov

cp. autoreg

20 Feb 2003

## Examples

**AR(1)** $Y_t = \varphi Y_{t-1} + Z_t$

$\{Z_t\} \sim WN(0, \sigma^2)$

$X_t = Y_t$

$G_t = 1$

$W_t = 0$

$F_t = \varphi$

## MA(1)

$$Y_t = Z_t + \theta Z_{t-1}$$

$$\underset{\sim}{X}_t = \begin{bmatrix} Y_t \\ Z_t \end{bmatrix}$$

$$\underset{\sim}{G} = [1, 0]$$

$$\underset{\sim}{F} = \begin{bmatrix} 0 & \theta \\ 0 & 0 \end{bmatrix}$$

$$W_t = 0$$

(5A) 20 Feb. 2003

# ARMA(1,1).

$$Y_t = \varphi Y_{t-1} + Z_t + \theta Z_{t-1}$$

$$\underset{\sim}{X}_t = X_{t-1}$$

 20 Feb 2003

## AR(p)

$$Y_{t+1} = \varphi_1 Y_t + \varphi_2 Y_{t-1} + \dots + \varphi_p Y_{t-p} + Z_{t+1}$$

$$\underset{\sim}{X}_t = \begin{bmatrix} Y_{t-p+1} \\ Y_{t-p+2} \\ \vdots \\ Y_t \end{bmatrix}$$

$$Y_t = \begin{bmatrix} 0 & \dots & 0 & 1 \end{bmatrix} X_t$$

$$\underset{\sim}{X}_{t+1} = \begin{bmatrix} 0 & 1 & 0 & & \\ 0 & 0 & 1 & & \\ \vdots & & & & \\ 0 & 0 & 0 & \dots & 1 \\ \varphi_p & \varphi_{p-1} & \varphi_{p-2} & \dots & \varphi_1 \end{bmatrix} \underset{\sim}{X}_t + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} Z_{t+1}$$

7    20 Feb. 2003

# Random walk plus noise

$$M_{t+1} = M_t + V_t \quad \{V_t\} \sim WN(0, \sigma_v^2)$$

$$Y_t = M_t + W_t \quad \{W_t\} \sim WN(0, \sigma_w^2)$$

$$X_t = M_t$$

 20 Feb 2003

## Locally $\left\{\begin{matrix} \text{drift} \\ \text{linear} \end{matrix}\right.$ trend

$$M_t = M_{t-1} + B_{t-1} + V_{t-1}$$

$$B_t = B_{t-1} + U_{t-1}$$

$$Y_t = M_t + W_t$$

$$\underset{\sim}{X}_t = \begin{bmatrix} M_t \\ B_t \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \underset{\sim}{X}_{t-1} + \underset{\sim}{V}_{t-1}$$

$$Y_t = \begin{bmatrix} 1 & 0 \end{bmatrix} X_t + W_t$$