Chapter 1, second half. Statistics 248 - Spring 2003 - D. R. Brillinger

$$E(aX + bY + c) \;=\; aE(X) + bE(Y) + c \tag{1}$$

$$Cov(aX + bY + c, Z) \;=\; aCov(X, Z) + bCov(Y, Z) \tag{2}$$

Linear filter:

$$\{Y_t\} \;=\; \{\sum_{j=-\infty}^{\infty} a_j X_{t-j}\} \tag{3}$$

E.g. running mean,

$$a_j \;=\; \frac{1}{2J+1} \sum_{j=-J}^{J} X_{t-j} \tag{4}$$

Backward shift operator:

$$BX_t \;=\; X_{t-1} \tag{5}$$

Lag-1 difference operator:

$$\bigtriangledown \;=\; X_t \;-\; X_{t-1} \;=\; (1 - B)X_t \tag{6}$$

Seasonal "removal",

$$X_t - X_{t-d} \;=\; (1 - B^d)X_t \tag{7}$$

Some acf's:
$Z \;\sim\; WN(0, \sigma^2)$,

$$\rho(s, t) \;=\; \delta_s^t \tag{8}$$

Random walk,

$$min(s, t)/\sqrt{st} \tag{9}$$

MA(1), $X_t \;=\; Z_t + \theta Z_{t-1}$

$$\begin{array}{cc} 1 & s = t \\ \theta/(1 + \theta^2) & s - t = \pm 1 \\ 0 & \end{array} \tag{10}$$

AR(1), $X_t \;=\; \phi X_{t-1} + Z_t, \quad |\phi| < 1$

$$\phi^{|s-t|} \tag{11}$$

Fitting an AR(1),

$$\min_{\phi} \;\sum_t \; (y_t \;-\; \phi y_{t-1})^2 \tag{12}$$

Assessing white noise,

$$\sqrt{n}\hat{\rho}(j) \;\approx\; IN(0, 1) \tag{13}$$

Portemanteau,

$$Q \;=\; n(n + 2) \sum_{j=1}^{h} \hat{\rho}^2(j)/(n - j) \;\approx\; \chi_h^2 \tag{14}$$