The S-method, \textbf{If}

\[
\begin{align*}
\hat{\mu}_n &= (\hat{\mu}_1, \ldots, \hat{\mu}_d) \\
\hat{\lambda} &= (\hat{\lambda}_1, \ldots, \hat{\lambda}_d) \\
\hat{\beta} &\sim N_d(\mu, \text{diag}(\frac{\sigma^2}{m_1}, \ldots, \frac{\sigma^2}{m_d})) \\
&\quad \text{for large } m_1, \ldots, m_d
\end{align*}
\]

and

\[\hat{\beta} = h(\mu), \ h \text{ differentiable}
\]

then

\[\hat{\beta} \sim N(\hat{\beta}, \text{AV}(\hat{\beta}))
\]

where

\[
\text{AV}(\hat{\beta}) = \sum_k \left( \frac{\partial h}{\partial \mu_k} (\mu) \right)^2 \frac{\sigma_k^2}{m_k}
\]

\textbf{Proof. Via Taylor series and convergence in probability.} \ (\hat{\mu}_k \xrightarrow{P} \mu_k)

\[
\text{ASD} = \sqrt{\frac{\text{AV}}{\sum_k \left( \frac{\partial h}{\partial \mu_k} (\mu) \right)^2 \frac{\sigma_k^2}{m_k}}}
\]

\[
\text{SE}(\hat{\beta}) = \sqrt{\frac{\sum_k \left( \frac{\partial h}{\partial \mu_k} (\mu) \right)^2 \frac{\sigma_k^2}{m_k}}}
\]
Two sample parameters

\[ B(m_1, \pi_1) \parallel B(m_2, \pi_2) \]

Difference: \( \pi_2 - \pi_1 = \Delta \)

Relative risk: \( \frac{\pi_2}{\pi_1} = \delta_{rr} \)

Odds ratio: \( \frac{\pi_2/(1-\pi_2)}{\pi_1/(1-\pi_1)} = \delta_{or} \)

\[ SE(\hat{\delta}_d) = \sqrt{\frac{\hat{\pi}_1(1-\hat{\pi}_1)}{n_1} + \frac{\hat{\pi}_2(1-\hat{\pi}_2)}{n_2}} \]

\[ SE(\hat{\delta}_{rr}) = \hat{\delta}_{rr} \sqrt{\frac{1 - \hat{\pi}_1}{n_1 \hat{\pi}_1(1-\hat{\pi}_1)} + \frac{1 - \hat{\pi}_2}{n_2 \hat{\pi}_2(1-\hat{\pi}_2)}} \]

\[ SE(\hat{\delta}_{or}) = \hat{\delta}_{or} \sqrt{\frac{1}{n_1 \hat{\pi}_1(1-\hat{\pi}_1)} + \frac{1}{n_2 \hat{\pi}_2(1-\hat{\pi}_2)}} \]
Abortion attitudes

Male vs. female attitudes

M support legal $y_1 = 319$, $m_1 = 600$

F " “ $y_2 = 309$, $m_2 = 500$

$H_0: \pi_1 = \pi_2 = 0 = \pi_0$

$\hat{\pi}_1 = 0.532 \quad \hat{\pi}_2 = 0.618 \quad \chi^2 = -0.086$

$\text{SE}(\hat{\pi}) = \sqrt{\frac{\hat{\pi}_1(1-\hat{\pi}_1)}{m_1} + \frac{\hat{\pi}_2(1-\hat{\pi}_2)}{m_2}}$

$\quad \quad \quad \quad = 0.030$

Wald statistic $W = \frac{(\hat{\pi} - \pi_0)}{\text{SE}(\hat{\pi})} = -2.87$

P-value $2[1 - F(1W)] = 0.0042$

$H_A: \pi \neq 0$

Infer women more likely to support
Relative risk: $\frac{\hat{\pi}_2}{\hat{\pi}_1}$

$H_0: \pi = \frac{\hat{\pi}_2}{\hat{\pi}_1} = 1 = \pi_0$

$\hat{\pi}_2 = \frac{\hat{\pi}_2}{\hat{\pi}_1} = 1.162$

$SE(\hat{\pi}) = \sqrt{\frac{1-\hat{\pi}_1}{n_1 \cdot \hat{\pi}_1} + \frac{1-\hat{\pi}_2}{n_2 \cdot \hat{\pi}_2}}$

$= 0.0604$

$W = \frac{(\hat{\pi} - \pi_0)}{SE(\hat{\pi})} = 2.677$

$P\text{-value: } 2 \left[ 1 - \Phi(1.677) \right] = 0.0074$
Grandchildren

\[ n = 3 \]

\[ y_d = 2 \text{ girls} \]

\[ H_0: \quad \pi = 1/2 \]

**P-value:**

\[ \text{Prob}\{ Y > 2 \} = \frac{\text{Prob}\{ \text{GGG, GGB, GGG} \}}{\text{Prob}\{ \text{GGB, GGG} \}} \]

\[ = \binom{3}{2}(\frac{1}{2})^2(\frac{1}{2}) + (\frac{1}{2})^3 \]

\[ = \frac{4}{8} \]
Exact P-values.

Binomial cdf

\[ \text{Prob}\{ Y \leq y \} = F(y; n, \pi) \]

\[ = \sum_{z=0}^{y} f(z; n, \pi) \]
Table 41. Chart providing confidence limits for \( p \) in binomial sampling, given a sample fraction \( c/n \).
Confidence coefficient, \( 1 - 2\alpha = 0.95 \).

The numbers printed along the curves indicate the sample size \( n \). If for a given value of the abscissa \( c/n \), \( p_1 \) and \( p_2 \) are the ordinates read from (or interpolated between) the appropriate lower and upper curves, then

\[
Pr(p_1 \leq p \leq p_2) = 1 - 2\alpha.
\]

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The Poisson: $P(\lambda)$

$Y = 0, 1, 2, \ldots$

$$P[Y = y] = \frac{\lambda^y}{y!} e^{-\lambda}$$

$$E(Y) = \lambda, \quad \text{var}(Y) = \lambda$$

$$P(A_1) + P(A_2) + \ldots + P(A_n) \overset{\text{independent}}{\sim} P(\lambda_1 + \lambda_2 + \ldots + \lambda_n)$$

$$B(m, \frac{\lambda}{m}) \rightarrow P(\lambda) \quad \text{as large rare events}$$

$$P(\lambda) \sim N(\lambda, \sqrt{\lambda}) \quad \text{as large}$$
Poisson $d$-sample model.

$Y_1, \ldots, Y_d \sim \text{IP}(\lambda_k)$

Hypothesis of homogeneity

$H_0: \lambda_1 = \lambda_2 = \ldots = \lambda_d$

Chapter 13
Boy's born
1334 Swedish ministers
No 0 1 2 3 4 5
Freq 6 54 206 362 365 256

6 7
69 13
Boys born

![Histogram showing the number of boys born across different categories.](image)