14 April 03

The δ-method

If
$$\underset{\sim}{\mu} = (\mu_1, \dots, \mu_d)$$
$$\hat{\underset{\sim}{\mu}} = (\hat{\mu}_1, \dots, \hat{\mu}_d)$$
$$\hat{\underset{\sim}{\mu}} \sim N_d\left(\underset{\sim}{\mu}, \operatorname{diag}\left(\frac{\sigma_1^2}{n_1}, \dots, \frac{\sigma_d^2}{n_d}\right)\right)$$
for large $n_1, \dots, n_d$

and
$$\tau = h(\underset{\sim}{\mu}), \quad h \text{ differentiable}, \ \nabla h \neq 0$$

then
$$\hat{\tau} \sim N(\tau, AV(\hat{\tau}))$$

where
$$AV(\hat{\tau}) = \sum_k \left(\frac{\partial h}{\partial \mu_k}(\underset{\sim}{\mu})\right)^2 \frac{\sigma_k^2}{n_k}$$

Proof. Via Taylor series and convergence in probability. ($\hat{\mu}_k \xrightarrow{p} \mu_k$)

$$ASD = \sqrt{AV}$$
$$SE(\hat{\tau}) = \sqrt{\sum_k \left(\frac{\partial h}{\partial \mu_k}(\hat{\mu})\right)^2 \frac{\hat{\sigma}_k^2}{n_k}}$$

14 April 03

Two sample parameters

$$B(n_1, \pi_1) \perp\!\!\!\perp B(n_2, \pi_2)$$

Difference: $\pi_2 - \pi_1 = \theta_d$

Relative risk: $\pi_2 / \pi_1 = \theta_{rr}$

Odds ratio: $\dfrac{\pi_2 / (1-\pi_2)}{\pi_1 / (1-\pi_1)} = \theta_{or}$

$$SE(\hat{\theta}_d) = \sqrt{\frac{\hat{\pi}_1(1-\hat{\pi}_1)}{n_1} + \frac{\hat{\pi}_2(1-\hat{\pi}_2)}{n_2}}$$

$$SE(\hat{\theta}_{rr}) = \hat{\theta}_{rr}\sqrt{\frac{1-\hat{\pi}_1}{n_1\hat{\pi}_1} + \frac{1-\hat{\pi}_2}{n_2\hat{\pi}_2}}$$

$$SE(\hat{\theta}_{or}) = \hat{\theta}_{or}\sqrt{\frac{1}{n_1\hat{\pi}_1(1-\hat{\pi}_1)} + \frac{1}{n_2\hat{\pi}_2(1-\hat{\pi}_2)}}$$

 14 April 03

Abortion attitudes

Male vs. female attitudes

M support legal $y_1 = 319$, $n_1 = 600$

F " " $y_2 = 309$, $n_2 = 500$

$H_0: \pi_1 - \pi_2 = 0 = \tau_0$

$\hat{\pi}_1 = .532$ $\hat{\pi}_2 = .618$ $\hat{\tau} = -.086$

$$SE(\hat{\tau}) = \sqrt{\frac{\hat{\pi}_1(1-\hat{\pi}_1)}{n_1} + \frac{\hat{\pi}_2(1-\hat{\pi}_2)}{n_2}}$$

$$= .030$$

Wald statistic $W = \dfrac{(\hat{\tau} - \tau_0)}{SE(\hat{\tau})} = -2.87$

P-value $2[1 - \Phi(|W|)] = .0042$

$H_A: \tau \neq 0$

Infer women more likely to support

14 April 03

Relative risk: π_2 / π_1

H_0: $\tau = \pi_2 / \pi_1 = 1 = \tau_0$

$$\hat{\tau} = \hat{\pi}_2 / \hat{\pi}_1 = 1.162$$

$$SE(\hat{\tau}) = \hat{\tau} \sqrt{\frac{1-\hat{\pi}_1}{n_1 \hat{\pi}_1} + \frac{1-\hat{\pi}_2}{n_2 \hat{\pi}_2}}$$

$$= .0604$$

$$W = \frac{(\hat{\tau} - \tau_0)}{SE(\hat{\tau})} = 2.677$$

P-value: $2[1 - \Phi(|W|)] = .0074$

14 April 03

Grandchildren

$n = 3$

$y = 2$ girls

$H_0 : \pi = 1/2$

P-value. $\text{Prob}\{Y \geq 2\} =$

$\text{Prob}\{GGB, GGG\}$

$$= \binom{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^3$$

$$= \frac{4}{8}$$

14 April 03

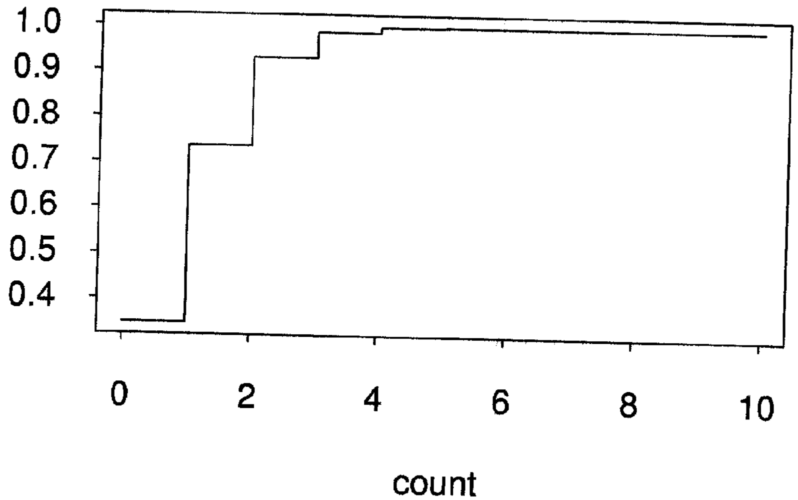
Exact P-values

Binomial cdf

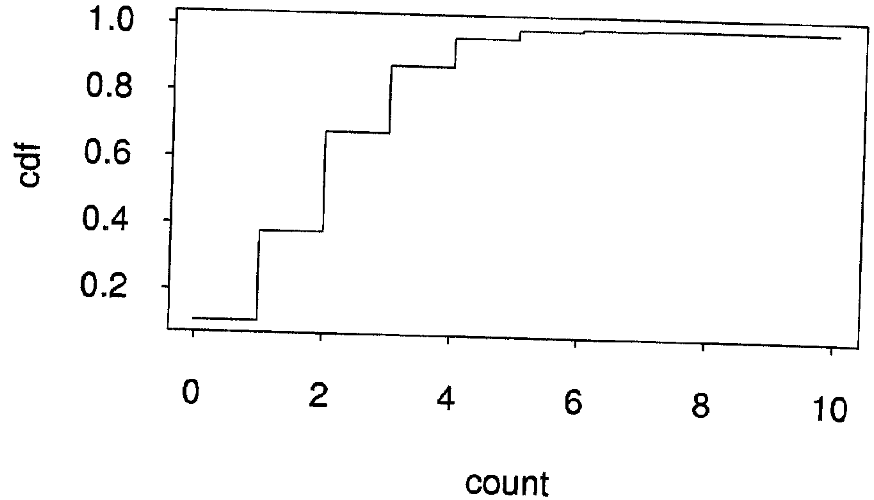
$$\text{Prob}\{Y \le y\} = F(y; n, \pi)$$

$$= \sum_{z=0}^{y} f(z; n, \pi)$$

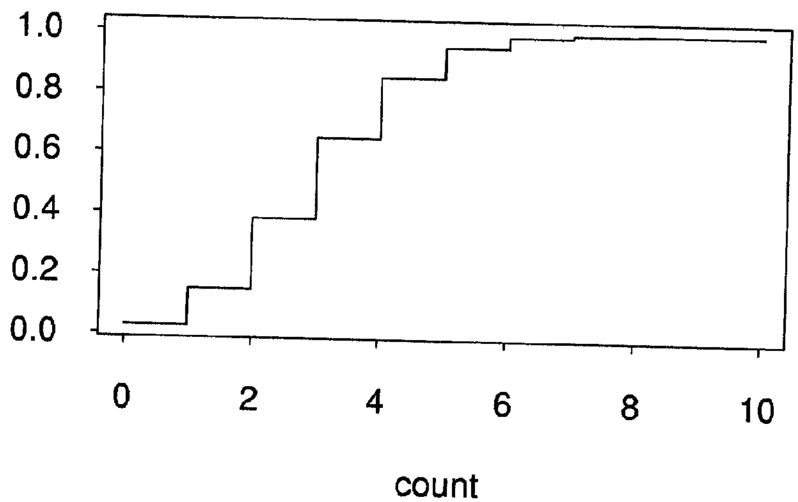
Binomial cdf, n = 10, param = 0.1



Binomial cdf, n = 10, param = 0.2



Binomial cdf, n = 10, param = 0.3



Binomial cdf, n = 10, param = 0.4

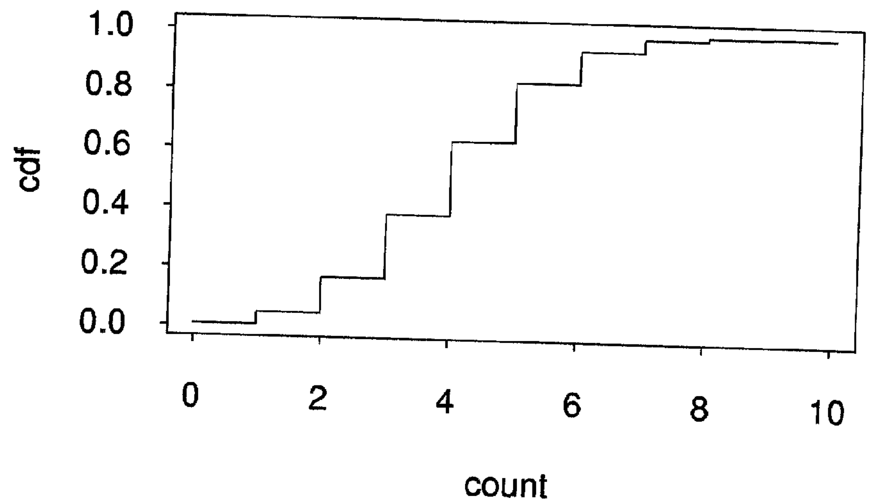
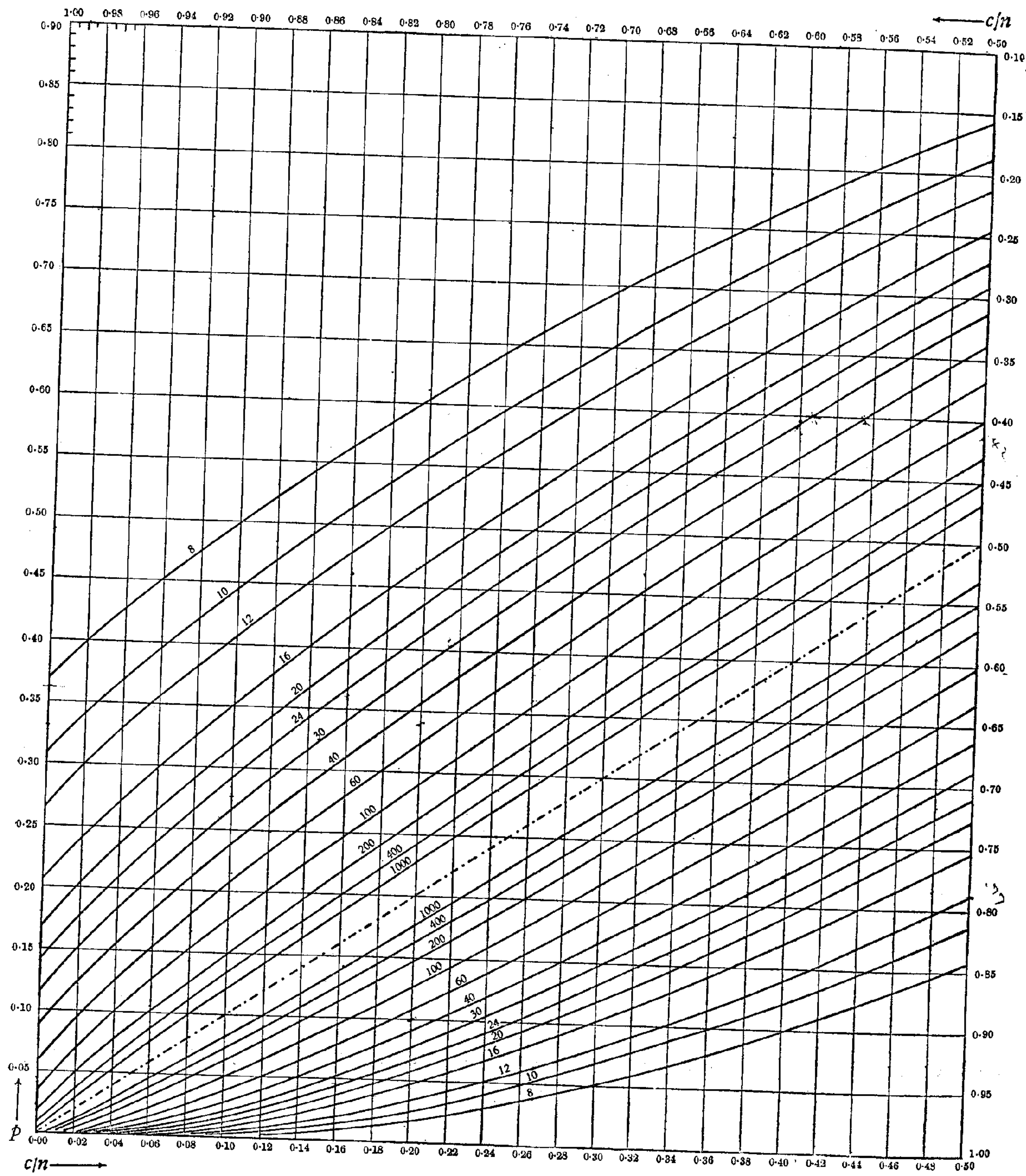


Table 41. *Chart providing confidence limits for p in binomial sampling, given a sample fraction c/n. Confidence coefficient,* $1-2\alpha=0{\cdot}95$.



The numbers printed along the curves indicate the sample size n. If for a given value of the abscissa c/n, p_A and p_B are the ordinates read from (or interpolated between) the appropriate lower and upper curves, then

$$\Pr\{p_A \leqslant p \leqslant p_B\} \geqslant 1-2\alpha.$$

The Poisson $P(\lambda)$

$Y = 0, 1, 2, \ldots$

$$\text{Prob}\{Y = y\} = \frac{\lambda^y}{y!} e^{-\lambda}$$

$E(Y) = \lambda, \quad \text{var}(Y) = \lambda$

$P(\lambda_1) + P(\lambda_2) + \ldots + P(\lambda_n)$ independent

$\sim P(\lambda_1 + \lambda_2 + \ldots + \lambda_n)$

$B(n, \frac{\lambda}{n}) \longrightarrow P(\lambda)$ n large

rare events

$P(\lambda) \sim N(\lambda, \sqrt{\lambda})$ λ large

14 April 03

Poisson d-sample model.

$Y_1, \dots, Y_d \sim IP(\lambda_k)$

Hypothesis of homogeneity

$H_0: \lambda_1 = \lambda_2 = \dots = \lambda_d$

Chapter 13

(12)

14 April 02

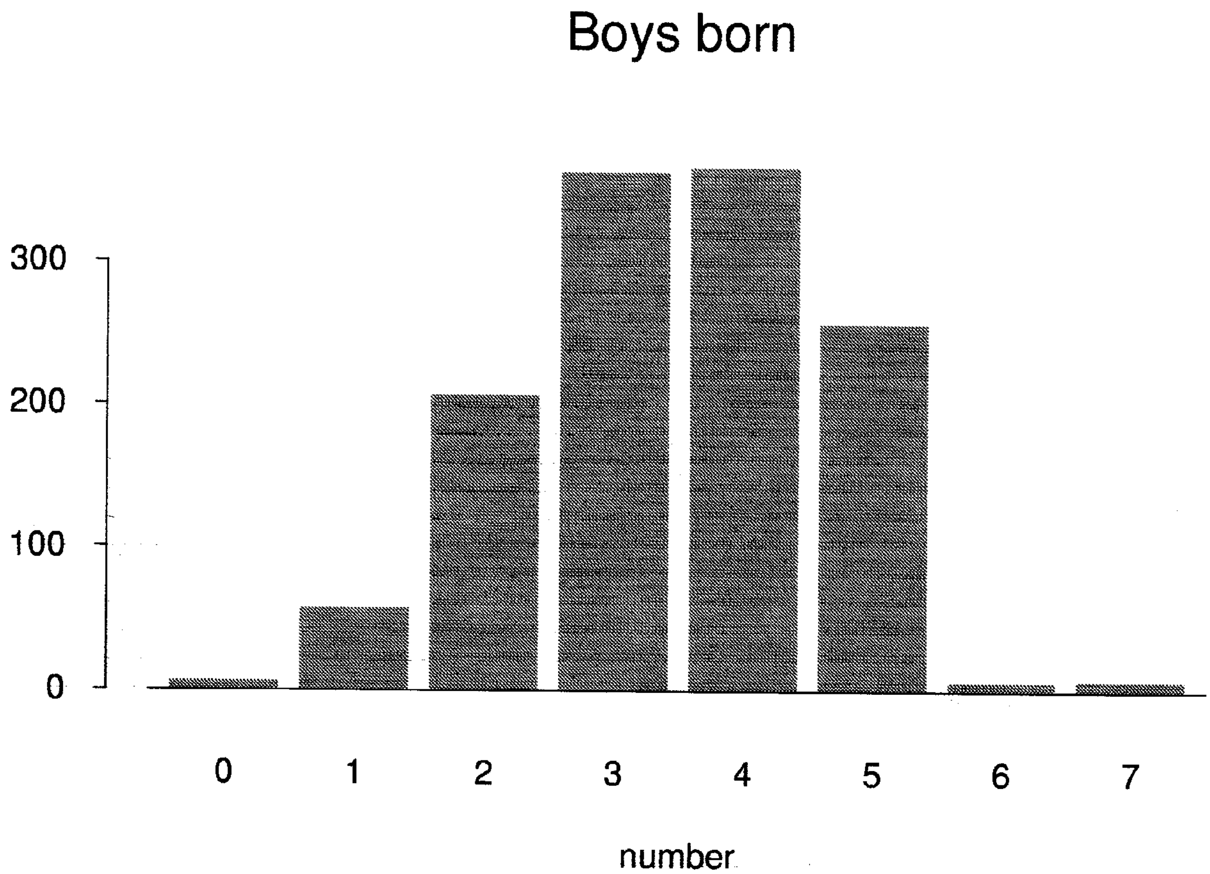
Boy's born

1334 Swedish ministers

No	0	1	2	3	4	5
Freq	6	57	206	362	365	256

6 7

69 13


Boys born
300
200
100
0
0
1
2
3
4
5
6
7
number