Homework 1.

1. (Problem 1.4 in Brockwell and Davis) Let $\{Z_t\}$ be a sequence of independent normal random variables, each with mean 0 and variance σ^2, and let a, b, and c be constants. Which, if any, of the following processes are stationary? For each stationary process specify the mean and autocovariance function.

a. $X_t \ = \ a + bZ_t + cZ_{t-2}$
b. $X_t \ = \ Z_1 cos(ct) + Z_2 sin(ct)$
c. $X_t \ = \ Z_t cos(ct) + Z_{t-1} sin(ct)$
d. $X_t \ = \ a + bZ_0$
e. $X_t \ = \ Z_0 cos(ct)$
f. $X_t \ = \ Z_t Z_{t-1}$

2. (Problem 1.7 in B and D). If $\{X_t\}$ and $\{Y_t\}$ are uncorrelated stationary sequences, i.e. if X_r and Y_s are uncorrelated for every r and s, show that $\{X_t + Y_t\}$ is stationary with autocovariance function equal to the sum of the autocovariance functions of $\{X_t\}$ and $\{Y_t\}$.

3. (Problem 1.11 in B and D). Consider the simple moving-average filter with weights $a_j \ = \ (2q+1)^{-1}, \ -q \leq j \leq q$.

a. If $m_t \ = \ c_0 + c_1 t$, show that $\sum_{j=-q}^{q} a_j m_{t-j} \ = \ m_t$.

b. If $Z_t, \ t = 0, \ \pm 1, \ \pm 2, ...,$ are independent random variables with mean 0 and variance σ^2, show that the moving average $A_t \ = \ \sum_{j=-q}^{q} a_j Z_{t-j}$ is "small" for large q in the sense that $EA_t \ = \ 0$ and $Var\ (A_t) \ = \ \sigma^2/(2q+1)$.