Chapter 7: Normal Models

\[ J = c_1 \mu_1 + \ldots + c_d \mu_d \]

Chapter 8: Intro to Linear Regression

\[ \mu(x) = \beta_0 + \beta_1 x \]
\[ \mu(x) = \beta_0 + \beta_1 x + \beta_2 x^2 \]
\[ \mu(x) = \beta_0 + \beta_1 x + \ldots + \beta_5 x^5 \]

\[ \mathcal{A} : \text{space of polynomials of degree } \leq p-1 \]
\[ \text{basis: } 1, x, \ldots, x^{p-1} \]
\[ g \in \mathcal{A}, g = b_0 + b_1 x + \ldots + b_{p-1} x^{p-1} \]
\[ \dim(\mathcal{A}) = p \]

\[ \mathcal{X}_m = \{ \text{all allowable levels of } m\text{th factor} \} \]
\[ \mathcal{X} = \mathcal{X}_1 \times \ldots \times \mathcal{X}_M \]
\[ d = \text{number of distinct factor level combinations} \]
\[ \mathcal{X}' = \{ x_1', \ldots, x_{d'} \} \quad \text{design set} \]

2 May 00
G: Hilbert space of functions on X

\[ \dim(G) = p \]

basis \[ g_1, \ldots, g_p \]

\[ \mu \in G: \quad \mu(x) = \beta_1 g_1(x) + \ldots + \beta_p g_p(x) \]

additive
\[ x = (x_1, \ldots, x_m) \]
\[ \mu(x) = \beta_1 g_1(x_1) + \ldots + \beta_m g_m(x_m) \]

interaction
\[ \text{eg.} \quad \beta_{12} g_1(x_1) g_2(x_2) \]

a sample model
\[ y_{ki} \quad 1 \leq k \leq d, 1 \leq i \leq m \]
\[ \mu(x) = \mu_1 I_1(x) + \ldots + \mu_d I_d(x) \]
\[ x = (x_1, \ldots, x_d) \]
\[ I_k(x) = \text{ind } (x = k) \]
\[ \mu(0) \in \mathcal{G} \]

Known as design points \( x_1', \ldots, x_d' \in \mathcal{X} \).

\( \mathcal{G} \) is identifiable (relative to the design set), if knowing \( \mu(x_1'), \ldots, \mu(x_d') \) determines \( \mu(0) \).

\( \mathcal{G} \) is identifiable iff the only \( g \in \mathcal{G} \) that equals 0 on \( x' \) is \( g \equiv 0 \).

Let \( X \) be the design matrix corresponding to a basis of functions on \( \mathcal{X} \). Then \( \mathcal{G} \) is identifiable iff \( X \) is nonsingular.