Randomized blocks

Manufacturing penicillin

A raw material (corn liquor) highly variable

Split into 5 blocks (blends) \( j = 1, \ldots, 5 \)
of 4 units

4 treatments (variants of process) \( i = 1, \ldots, 4 \)

Applied randomly (time order)

Response: yield (amount)
Additive

\[ f = y + \beta + \gamma + \epsilon \]

\[ f_i = y_i + \beta_i + \gamma_i + \epsilon_i \]

\[ f_i = y + \beta + \gamma + \epsilon \]

Model

\[
\begin{array}{c|cccc|c|c}
\text{Block} & 86 & 88 & 89 & 88 & \text{Mean} \\
8 & 86 & 88 & 88 & 86 & 86 \\
88 & 88 & 88 & 88 & 88 & 88 \\
88 & 88 & 88 & 88 & 88 & 88 \\
88 & 88 & 88 & 88 & 88 & 88 \\
88 & 88 & 88 & 88 & 88 & 88 \\
88 & 88 & 88 & 88 & 88 & 88 \\
88 & 88 & 88 & 88 & 88 & 88 \\
88 & 88 & 88 & 88 & 88 & 88 \\
88 & 88 & 88 & 88 & 88 & 88 \\
\text{Mean} & 86 & 88 & 89 & 88 & 88 \\
\end{array}
\]

\[ A \quad B \quad (\text{Within}) \quad (\text{Between}) \quad \text{Block} \]

Data

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\[
\begin{array}{c|c|c|c|c}
\text{1} & \text{2} & \text{3} & \text{Total} & \text{Residuals} \\
\text{Total} & \text{3} & \text{3} & \text{3} & \text{3} \\
\text{Blends} & \text{3} & \text{3} & \text{3} & \text{3} \\
\text{Treatments} & \text{3} & \text{3} & \text{3} & \text{3} \\
\text{Source} & \text{3} & \text{3} & \text{3} & \text{3} \\
\text{Error} & \text{3} & \text{3} & \text{3} & \text{3} \\
\text{Grand Mean} & \text{3} & \text{3} & \text{3} & \text{3} \\
\end{array}
\]
\[ \frac{84 - 85}{1.149 (4.329)} = \frac{84 - 85}{5.96} = 100 + 5.96 \]

\[ 84 - 85 = \frac{1.149 (4.329)}{5.96} \frac{84 - 85}{1.149} \]

\[ \frac{84 - 85}{1.149} = \frac{1.149 (4.329)}{5.96} \]

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\[ \text{Comparing treatment means} \]

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be applicable to which treatment combination can be applied.

Experimental unit, smallest unit, 31 March 03 ⑤
R = no. units / block
B = no. blocks
f = no. treatments each block
n = no. replicates each treatment
n x f = b x k

<table>
<thead>
<tr>
<th>Block</th>
<th>Treat 1</th>
<th>Treat 2</th>
<th>Treat 3</th>
<th>Treat 4</th>
<th>Treat 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Blocks:

Rows:

1. Each treatment same number of times
2. Any 2 treatments occur together in one block
3. Each treatment same number of times

31 March 03
Design of experiments

- Want $\mathbf{X}$ large

$\text{Var}(\mathbf{X}) = \mathbf{X} \cdot \mathbf{X}^T$

$\mathbf{I} = \mathbf{X} \cdot \mathbf{X}^T$

How to choose $\mathbf{X}$?

$\text{Var}(\mathbf{X}) = \mathbf{I}$

$\mathbf{Y} = \mathbf{X} + \mathbf{w}$

Orthogonality

31 March 2003

1
and its Applications

C.R. Rao (1973), Linear Statistical Inference

with min when \( \bar{X} = 0 \), if

\[ \frac{3}{2} \leq \frac{1}{C^2} \]

then

\[ \Pr(\bar{X} \sim \bar{X}) = 0 \]

and

\[ \left[ \begin{array}{c}
0 \\
\vdots \\
0
\end{array} \right] = \left[ \begin{array}{c}
C^2 \\
\vdots \\
C^2
\end{array} \right] \]

Proof for orthogonal \( \bar{X} \) columns of \( \bar{X} \).

with min when \( \bar{X} = 0 \), if

\[ \text{Var} (\bar{X}) \approx \frac{1}{C^2} \]

then

\[ \bar{X}_j \sim \text{f-th} \text{ column of } \bar{X} \]

3rd March 03
\[ \frac{\tilde{1}_{x, \tilde{x}}/1_{\tilde{1}}}{} = 0 \cdot c \text{ where } C = 1 \]

\[ \text{Var}(\tilde{c}) = 0 \cdot c \]

and so clearly when \( b = 0 \), \( \tilde{c} = 0 \) as \( b \geq b' \) so \( \tilde{c} = 0 \).

\[ \tilde{x}' \tilde{x} \leq \tilde{x}' \tilde{x} - B \tilde{f} = \frac{1 \tilde{x} x}{1_{\tilde{1}}} = \tilde{x}' \tilde{x} \]

\[ (\tilde{x}' \tilde{x} = 1_{\tilde{1}} \cdot \tilde{x}' \tilde{x} = B \tilde{f}) \]

\[ \begin{bmatrix} 1 & \tilde{f} \\ \tilde{B} & \tilde{x}' \tilde{x} \end{bmatrix} = \tilde{x}' \tilde{x} \]

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<table>
<thead>
<tr>
<th></th>
<th>$X$</th>
<th>$Y$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
<td>$p$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Source</td>
<td>$I$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>SS</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$SS$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$H_{null}$</td>
<td></td>
</tr>
</tbody>
</table>

\[
\text{cov}(\bar{y}^i, \bar{x}^j) = 0, \quad j \neq i
\]

After partition

above

After centering

\[
\bar{y}^i = \bar{y}^i / \bar{x}
\]

computation

Advantages of orthogonality.