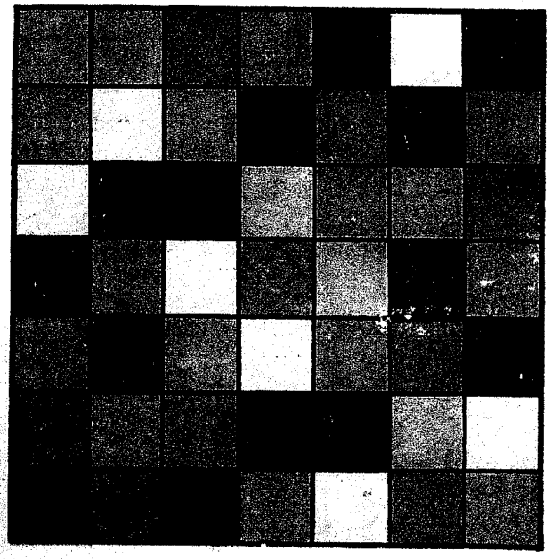
(0)

THE DESIGN OF EXPERIMENTS

FISHER

SEVENTH EDITION

THE DESIGN OF EXPERIMENTS



purple
red
green
pink
brown
yellow
blue

23 April 08

Latin square

Example. Field concrete mixer data

4 treatments: 4, 8, 12, 16 mph

response: efficiency

≡ ~~street~~ hardness / lab hardness

4 days of the week — blocking factors

4 runs/trials per day

$t(r,c)$	t: treatment	1:4	1:8
	r: day	1:4	1:8
	c: run	1:4	1:8

y_{rc}

Day \ Run	1	2	3	4
1	8	16	4	12
2	16	12	8	4
3	4	8	12	16
4	12	4	16	8

Entry: speed applied randomly

$$y_{rc} = \mu + \alpha_r + \beta_c + \gamma_{t(r,c)} + \varepsilon_{r,c}$$

$1 + 3q$ parameters

$$r(X) = 1 + 3(q-1)$$

Decomposition of response

$$y_{rc} = \bar{y}_{..} + (\bar{y}_{r.} - \bar{y}_{..}) + (\bar{y}_{.c} - \bar{y}_{..}) + (\bar{y}_{t} - \bar{y}_{..})$$
$$+ (y_{rc} - \bar{y}_{r.} - \bar{y}_{.c} - \bar{y}_{t} + 2\bar{y}_{..})$$

$$y_{rc} = \hat{\mu} + \hat{\alpha}_r + \hat{\beta}_c + \hat{\gamma}_t + \hat{\varepsilon}_{rc}$$

Anova Table

Term	df	SS	MS = SS/df	F
Rows	$q-1$	$\sum_{r,c} (\bar{y}_{r.} - \bar{y}_{..})^2$		
Columns	$q-1$	$\sum (\bar{y}_{.c} - \bar{y}_{..})^2$		
Treatments	$q-1$	$\sum (\bar{y}_{t(r,c)} - \bar{y}_{..})^2$		
Residual	$(q-1)(q-2)$	$\sum_{r,c} (y_{rc} - \bar{y}_{r.} - \bar{y}_{.c} - \bar{y}_{t(r,c)} + 2\bar{y}_{..})^2$		
Total	q^2-1	$\sum_{r,c} (\bar{y}_{rc} - \bar{y}_{..})^2$		

Observed responses

64.2 (8)	59.8 (16)	66.2 (4)	63.6 (12)
47.5 (16)	51.3 (12)	61.7 (8)	58.6 (4)
54.2 (4)	59.9 (8)	57.1 (12)	54.1 (16)
60.1 (12)	68.4 (4)	58.7 (16)	63.7 (8)

ANOVA TABLE

Term	df	SS	MS = SS/df	F = MS/MS$_R$
Days	3	151.06	50.35	5.78
Runs	3	79.74	26.58	3.05
Speeds	3	173.58	57.86	6.64
Residual	6	52.23	8.71	
Total				

$F_{3,6}(.95) = 4.578$

prob-values

pf(5.78, 3, 6) = .967 1 − pf = .0334

1 − pf(3.05, 3, 6) = .1138

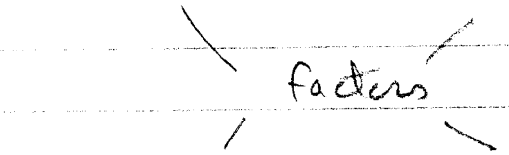
1 − pf(6.64, 3, 6) = .0247

Average speeds 61.85 (4) * 63.88 (8) 59.53 (12)
55.03 (16)

Factorial design

Poisons data Table 7.10 p. 391

3 poisons 4 ~~treatments~~ (universal terminology)



P1 P2 P3 T1 T2 T3 T4

3 levels 4 levels

12 combinations

48 animals (units) 4 replicates

randomly formed 12 groups of 4

each collection of 12 received all 12 combinations

factors: quantities that are varied over levels

factorial experiment: data are collected at each combination of a number of factors

Data responses lifetimes (10hr. units)

	P1	P2
T1	.31, .45, .46, .43	
T2	.82, 1.10, .88, .72	
T3	.43, .45, .63, .76	
T4	.45, .71, .66, .62	

P3

T1
T2
T3
T4

Model

Response: r.v.

y_{ttpj} : $t=1,2,3,4$ $p=1,2,3$ $j=1,2,3,4$

Model

$$y_{ttpj} = \mu + \alpha_t + \beta_p + \gamma_{tp} + \varepsilon_{tpj}$$

γ_{tp} : interaction

overparametrized

$$\sum_t \alpha_t , \sum_p \beta_p , \sum_t \gamma_{tp} , \sum_p \gamma_{tp} = 0$$

orthogonal decomposition

$$y_{tpj} - \bar{y}_{...} =$$

$$(\bar{y}_{t..} - \bar{y}_{...}) + (\bar{y}_{.p.} - \bar{y}_{...}) + (\bar{y}_{tp.} - \bar{y}_{t..} - \bar{y}_{.p.} + \bar{y}_{...})$$

$$+ (y_{tpj} - \bar{y}_{tp.})$$

Error in test re interaction

ANOVA Table

Term	df	SS
Rows	$T-1$	$\sum_{tpj} (\bar{y}_{t\cdot\cdot\cdot} - \bar{y}_{\cdot\cdot\cdot\cdot})^2$
Columns	$P-1$	$\sum_{tpj} (\bar{y}_{\cdot p\cdot\cdot} - \bar{y}_{\cdot\cdot\cdot\cdot})^2$
Rows x Columns	$(T-1)(P-1)$	$\sum (\bar{y}_{tp\cdot\cdot} - \bar{y}_{t\cdot\cdot\cdot} - \bar{y}_{\cdot p\cdot\cdot} + \bar{y}_{\cdot\cdot\cdot\cdot})^2$
Residual	$TP(J-1)$	$\sum (y_{tpj} - \bar{y}_{tp\cdot\cdot})^2$
Total	$TPJ-1$	

Parsons

Term	df	SS	$MS = \frac{SS}{df}$	$F = SS/SS_R$
Parsons	2	1.033	.517	23.22
Treatments	3	.921	.307	13.81
Treatments x Parsons	6	.250	.042	1.87
Residual	36	.801	.022	
Total	47			

1 - pf(23.22, 2, 36) = 3.334 E-07

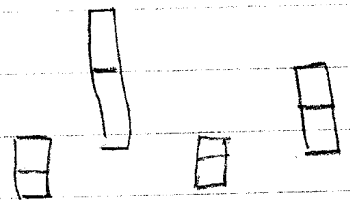
1 - pf(13.81, 3, 36) = 3.766 E-06

1 - pf(1.87, 6, 36) = .113

T_2, T_4 prolong life best

T_1 shortens it

Look at parallel boxplots page 392



Interpretation of interaction

Polls	Clinton	Obama	Margin error	n
Suffolk U	52	42	±4	600
Quinnipiac U	51	44	±3	1027
Mason-Dixon	48	43	±4	625
Result	55	45		

$\frac{10}{94} = .106$

$\frac{7}{95} = .074$

$\frac{5}{91} = .055$