21 April 08

Chap 9 Designed Experiments

One-way layout
Randomized blocks
Latin square
Factorial design
common designs and analysis

Interactions
Contrasts
Analysis of covariance

Components of variance
Nested variation
Split unit experiments
Linear mixed models

Experiment

<u>Units</u> - raw material under investigation

smallest subdivision such that any two different units might receive different treatments

<u>Treatments</u> - clearly defined procedures one of which is to be applied to each experimental unit

<u>Response</u> - measurement on unit after ~~treatment~~ applied

Example 9.2. Teaching methods data

45 pupils - units

Treatments - teaching methods

- A usual methods
- B " "
- C praised publicly
- D reproved " "
- E ignored

Response - test result

Group	Test result y				Average	Variance
A	17	14	24	...	19.67	17.75
B	21	23	13	...	18.33	12.75
C	28	30	29	...	27.44	6.03
D	19	28	26	...	23.44	9.53
E	21	14	13	...	16.11	13.11

$T = 5$ ~~treatments~~

$n = RT = 45$ units

T groups of size R $\quad R = 9$

Linear model

$$y_{tr} = \beta_t + \varepsilon_{tr} \quad t = 1, .., T \quad r = 1, .., R$$

$$\varepsilon_{tr} \sim IN(0, \sigma^2)$$

One-way layout

$$\begin{array}{cccccc} n \times 1 & & n \times T & T \times 1 & & n \times 1 \\ \begin{bmatrix} y_{11} \\ \vdots \\ y_{1R} \\ y_{21} \\ \vdots \\ y_{2R} \\ \vdots \\ y_{T1} \\ \vdots \\ y_{TR} \end{bmatrix} & = & \begin{bmatrix} 1 & 0 & \dots & 0 \\ 1 & 0 & \dots & 0 \\ \vdots & & & \\ 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & & & \\ 0 & 1 & \dots & 0 \\ & & & \\ 0 & 0 & \dots & 1 \\ \vdots & & & \\ 0 & 0 & \dots & 1 \end{bmatrix} & \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_T \end{bmatrix} & + & \begin{bmatrix} \varepsilon_{11} \\ \vdots \\ \varepsilon_{1R} \\ \varepsilon_{21} \\ \vdots \\ \varepsilon_{2R} \\ \vdots \\ \varepsilon_{T1} \\ \vdots \\ \varepsilon_{TR} \end{bmatrix} \end{array}$$

$$y = X\beta + \varepsilon$$

$$\hat{\beta}_t = \bar{y}_{t\cdot} = \sum_r y_{tr} / R$$

Decomposition

$$y_{tr} = \bar{y}_{\cdot\cdot} + \overline{\bar{y}_{t\cdot} - \bar{y}_{\cdot\cdot}} + \overline{y_{tr} - \bar{y}_{t\cdot}}$$

cp.

$$y_{tr} = \alpha + \gamma_t + \varepsilon_{tr} \qquad t = 1, .., T \quad r = 1, .., R$$

α: overall mean

$$\hat{\alpha} = \bar{y}_{\cdot\cdot}$$

γ_t: effect of ~~treatment~~ t

$$\sum_t \gamma_t = 0 \qquad \text{for estimability}$$

$$\hat{\gamma}_t = \bar{y}_{t\cdot} - \bar{y}_{\cdot\cdot}$$

ε_{tr}: error

Analysis of variance

$$\sum_{t,r} (y_{tr} - \bar{y}_{..})^2$$

$$= \underbrace{\sum_{t,r} (y_{tr} - \bar{y}_{t.})^2}_{\text{SS groups}} + \underbrace{\sum_{t,r} (\bar{y}_{t.} - \bar{y}_{..})^2}_{\text{SS residual}}$$

$p = T \quad \beta$'s

degrees of freedom $n - p = TR - T$

$$s^2 = (y - \hat{y})^T (y - \hat{y}) = \hat{\sigma}^2$$

$$= \frac{1}{T(R-1)} \sum_{t,r} (y_{tr} - \bar{y}_{t.})^2$$

H_0: $\beta_1 = \beta_2 = \ldots = \beta_T$

$\gamma_1, \ldots, \gamma_T = 0$

$$F = \frac{(T-1)^{-1} \sum_t (\bar{y}_{t.} - \bar{y}_{..})^2}{s^2}$$

$\sim F_{T-1,\, T(R-1)}$ under null

ANOVA Table

Term	df	SS	$MS = \frac{SS}{df}$	$F = \frac{MS_G}{MS_R}$
Groups	$T-1$	$\sum_{t,r}(\bar{y}_{t.} - \bar{y}_{..})^2$		
Residual	$T(R-1)$	$\sum_{t,r}(y_{tr} - \bar{y}_{t.})^2$		
Total	$TR-1$	$\sum_{t,r}(y_{tr} - \bar{y}_{..})^2$		

Term	df	SS	MS	F
Groups	4	722.67	180.67	15.3
Residual	40	473.33	11.83	
Total	45 - 1			

$F_{4,40}(.99) = 3.828$

P-value $= .9999999$

Residuals $y_{tr} - \bar{y}_{t.}$

Example 9.3

32 pigs - units

Treatments

Diet I

Diet II

Diet III

Diet IV

Response - weight gain

Diet	Group 1	Group 2	Average
I	1.40	1.79	1.48
II	1.31	1.30	1.21
III	1.40	1.47	1.33
IV	1.96	1.77	1.70
Average	1.52	1.58	1.43

$T = 4$ ~~treatments~~
$B = 8$ groups / blocks
similar units

$n = TB$

Linear model

$$y_{tb} = \mu + \alpha_t + \beta_b + \varepsilon_{tb}$$

$$t = 1, \dots, T \qquad b = 1, \dots, B$$

<u>Randomized block design</u>

T units applied randomly to each block/group

$\overset{n \times l}{X}$ rank $1 + (T-1) + (B-1)$

$\Sigma \alpha_t \,,\ \Sigma \beta_t = 0$

$$\underset{\sim}{\beta} = \begin{bmatrix} \mu \\ \alpha_1 \\ \vdots \\ \alpha_T \\ \beta_1 \\ \vdots \\ \beta_B \end{bmatrix}$$

Decomposition

$$y_{tb} = \bar{y}_{..} + \bar{y}_{t.} - \bar{y}_{..} + \bar{y}_{.b} - \bar{y}_{..}$$
$$+ y_{tb} - \bar{y}_{t.} - \bar{y}_{.b} + \bar{y}_{..}$$

$$\hat{\mu} = \bar{y}_{..}$$

$$\alpha_t = \bar{y}_{t.} - \bar{y}_{..}$$

$$\hat{\beta}_b = \bar{y}_{.b} - \bar{y}_{..}$$

Residual

$$\hat{\varepsilon}_{tb} = y_{tb} - \bar{y}_{t.} - \bar{y}_{.b} + \bar{y}_{..}$$

Analysis of Variance

$$\sum_{t,b}(y_{tb}-\bar{y}_{..})^2 = \sum_{t,b}(\bar{y}_{t.}-\bar{y}_{..})^2 + \sum_{t,b}(\bar{y}_{.b}-\bar{y}_{..})^2$$

$$+\sum_{t,b}(y_{tb}-\bar{y}_{t.}-\bar{y}_{.b}+\bar{y}_{..})^2$$

ANOVA Table

Term	df	SS	$MS = \frac{SS}{df}$	F
Treatments	$T-1$	$\sum_{t,b}(\bar{y}_{t.}-\bar{y}_{..})^2$		MS_T/MS_R
Blocks	$B-1$	$\sum_{t,b}(\bar{y}_{.b}-\bar{y}_{..})^2$		MS_B/MS_R
Residual	$(T-1)(B-1)$	$\sum_{t,b}(y_{tb}-\bar{y}_{t.}-\bar{y}_{.b}+\bar{y}_{..})^2$		
Total	$TB-1$	$\sum_{t,b}(y_{tb}-\bar{y}_{..})^2$		

Term	df	SS	MS	F
Diet	3	1.042	.347	14.6
Group	7	.247	.035	1.48
Residual	21	.500	.024	
Total	31			

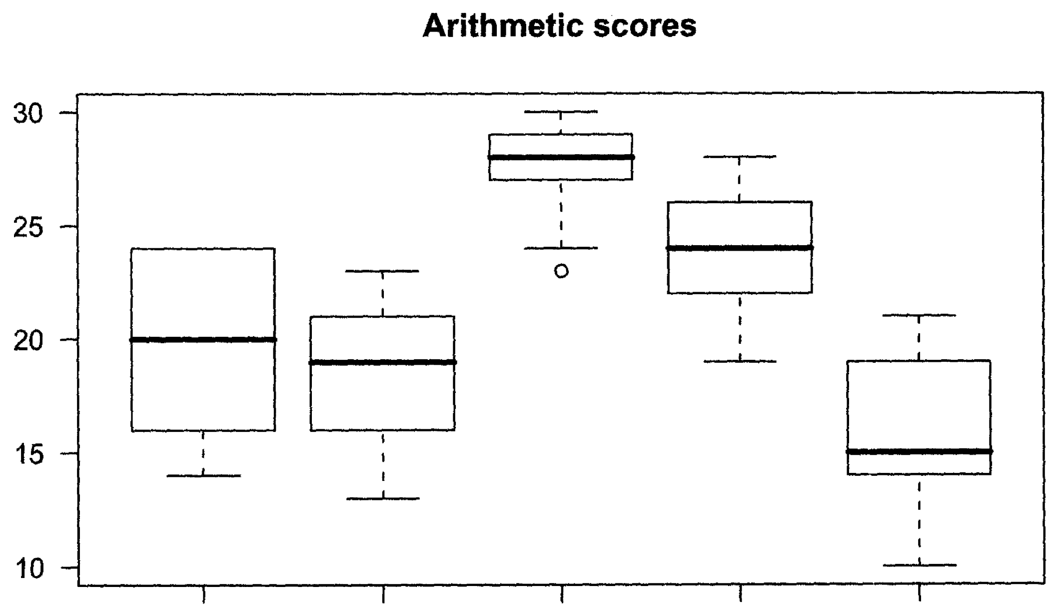
(13)

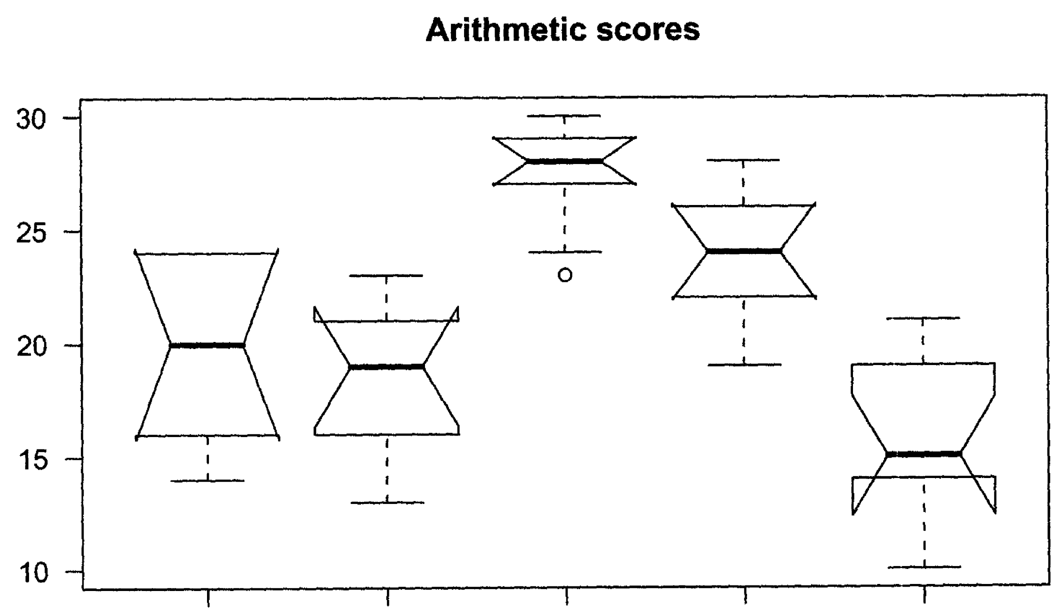
$F_{3,21}(.99) = 4.874$ 8f

$F_{7,21}(.99) = 3.640$

$pf(4.6, 3, 21) = .99997$

$pf(1.48, 7, 21) = .77205$


Arithmetic scores
30
25
20
15
10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