28 April 08

Designed Experiments

One-way layout

$$y_{tr} = \beta_t + \varepsilon_{tr} \qquad t=1,\ldots,T;\ r=1,\ldots,R$$
$$= \alpha + \gamma_t + \varepsilon_{tr}$$

Randomized block

$$y_{tb} = \mu + \alpha_t + \beta_b + \varepsilon_{tb} \quad t=1,\ldots,T;\ b=1,\ldots,B$$

Latin square

$$y_{rc} = \mu + \alpha_r + \beta_c + \gamma_{t(r,c)} + \varepsilon_{rc}$$
$$r=1,\ldots,q \quad ; \quad c=1,\ldots,q$$
$$t(r,c) = 1,\ldots,q$$

<u>Two factor experiment</u>
with replicates

$$y_{tpj} = \mu + \alpha_t + \beta_p + \gamma_{tp} + \varepsilon_{tpj}$$

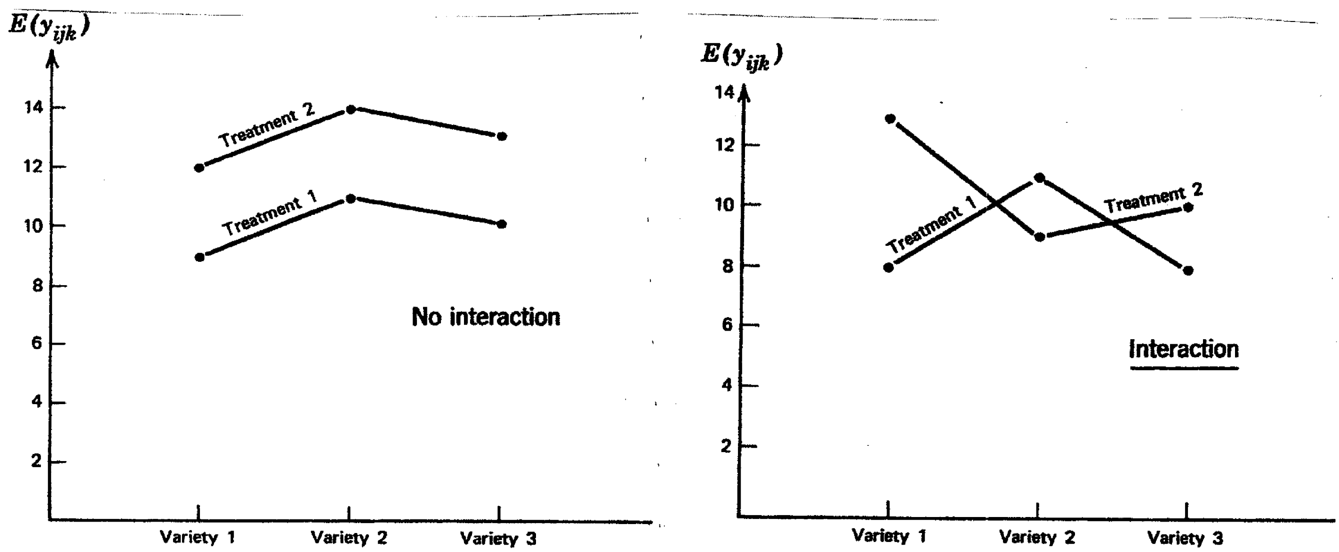
$t=1,\ldots,T$; $p=1,\ldots,P$; $j=1,\ldots,J$

all combinations of tp

$n = TPJ$

Interpretation of interaction

$$\bar{y}_{tp\cdot} - \bar{y}_{t\cdot\cdot} - \bar{y}_{\cdot p\cdot} + \bar{y}_{\cdots}$$



Additive $\equiv$ no interaction

Difference in $E\,y_{tpj}$ for t going

from t' to t'' is same for all p

Three factor experiment

$E\, y_{ijkl} =$

$$\alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk}$$

$$+ (\alpha\beta\gamma)_{ijk}$$

2^3 factorial experiment

3 factors A, B, C each with two levels
denoted -1 $+1$

Unit	Treatment	Intercept I	Main effects A	B	C	Two-factor interactions AB	AC	BC	Three-factor interaction ABC
1	1	+	−	−	−	+	+	+	−
2	*a*	+	+	−	−	−	−	+	+
3	*b*	+	−	+	−	−	+	−	+
4	*ab*	+	+	+	−	+	−	−	−
5	*c*	+	−	−	+	+	−	−	+
6	*ac*	+	+	−	+	−	+	−	−
7	*bc*	+	−	+	+	−	−	+	−
8	*abc*	+	+	+	+	+	+	+	+

Design matrix satisfies

$$X^T X = 8I$$

Effect estimates orthogonal.

$$\hat{\beta}_A \equiv (a-1)(b+1)(c+1)$$
$$= \frac{1}{8}\left(y_{abc} + y_{ab} + y_{ac} + y_a - y_{bc} - y_b - y_c - y_1\right)$$

$$\hat{\beta}_{BC} = (a+1)(b-1)(c-1)$$

bicycle data

Fractional replication

Factors $A, B, C, \ldots$

Number of runs $n = ABC\ldots$

Run a balanced subset

Example Latin square

q^2 runs not q^3