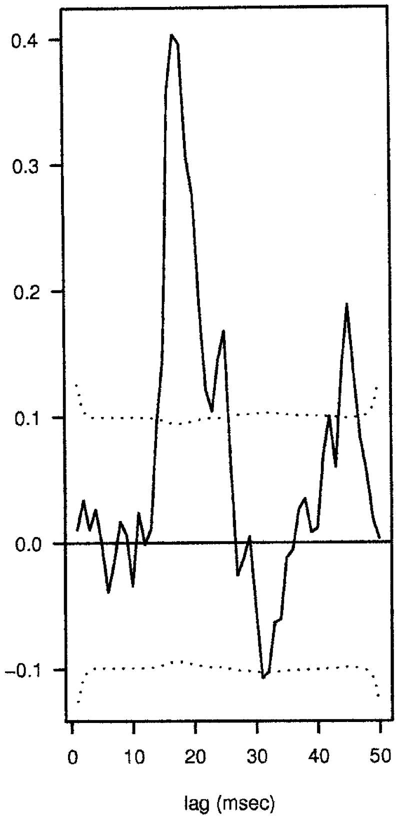
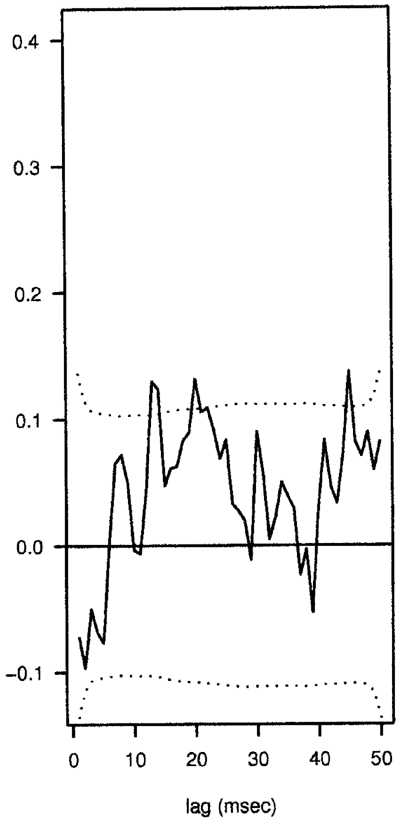
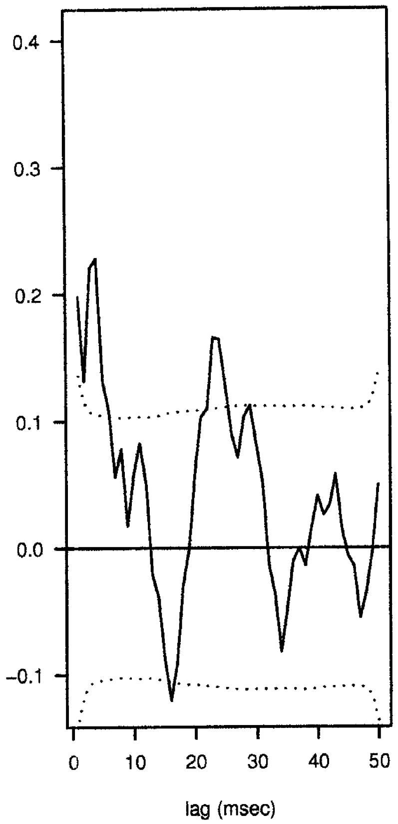


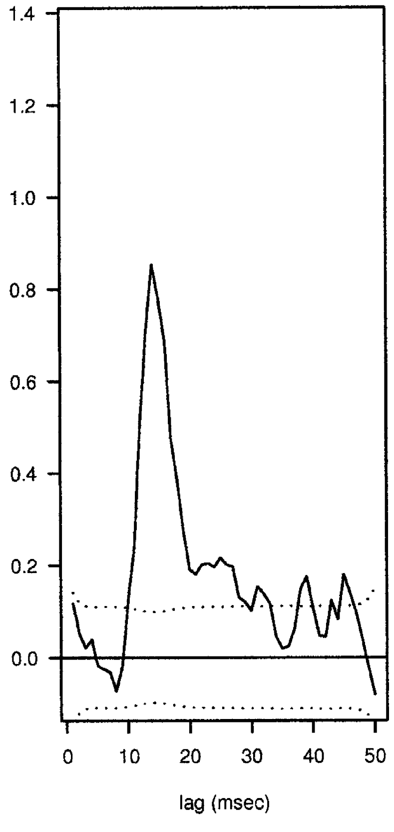
Il from Ia, g1, g2 – g1 function



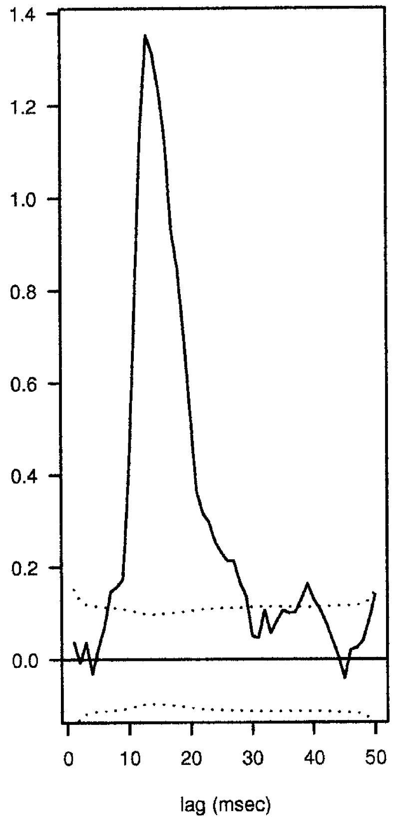
Il from Ia, g1, g2 – g2 function



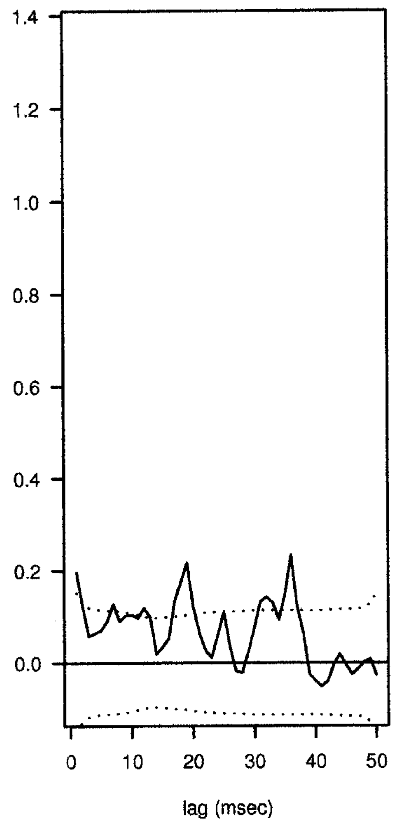
Il from Ia, g1, g2 – Ia function



Ia from Il, g1, g2 – g1 function



Ia from Il, g1, g2 – g2 function



Ia from Il, g1, g2 – Il function

16April08

Scaled deviance

$$D = 2\sum_{j=1}^{n}\{\log f(y_j;\tilde{\eta}_j) - \log f(y_j;\hat{\eta}_j)\}$$

Model fits well $\hat{\eta}_j$ and $\tilde{\eta}_j$ are close.

Normal deviance

$$\log f(y_j;\eta_j,\phi) = -\frac{1}{2}\log(2\pi\phi) - (y_j-\eta_j)^2/\phi$$

$$\tilde{\eta}_j = y_j$$

$$\log f(y_j;\tilde{\eta}_j,\phi) = -\frac{1}{2}\log 2\pi\phi$$

$$D = \frac{1}{\phi}\sum_{j=1}^{n}(y_j-\hat{\eta}_j)^2 \equiv \frac{1}{\phi}RSS$$

If $\eta_j = x_j^T\beta$, $RSS \sim \phi\chi^2_{n-p}$

$$\frac{RSS}{\phi} \sim \chi^2_{n-p}$$

Check of fit

β $p \times 1$

$H_0: \beta_{q+1}, \dots, \beta_p = 0$

Model B q β's free to vary

Model A p β's free to vary

B is nested within A

<u>Likelihood ratio statistic</u>

$$2 \sum_{j=1}^{n} \{ \log f(y_j ; \hat{\eta}_j^A) - \log f(y_j ; \hat{\eta}_j^B) \}$$

$$= D_B - D_A$$

Under H_0 χ^2_{p-q}

Normal case

$$D_B - D_A = \phi^{-1} \sum_{j=1}^{n} \{ (y_j - \hat{\eta}_j^B)^2 - (y_j - \hat{\eta}_j^A)^2 \}$$

$$\sim \chi^2_{p-q}$$

Logistic regression, neuro example

$$\Pr(Y=1) = \pi = \exp\{x^T\beta\}/(1+\exp\{x^T\beta\})$$

$$\Pr(Y=0) = 1-\pi = 1/(1+\exp\{x^T\beta\})$$

odds of success

$$\frac{\Pr(Y=1)}{\Pr(Y=0)} = \frac{\pi}{1-\pi} = \exp\{x^T\beta\}$$

log odds $= x^T\beta$

logit likelihood

$$L(\beta) = \prod_{j=1}^{n}\left\{\frac{\exp\{x^T\beta\}}{1+\exp\{x^T\beta\}}\right.$$

$$= \frac{\exp\{\Sigma_j\, y_j x_j^T\beta\}}{\Pi_j\{1+\exp\{x_j^T\beta\}\}}$$

$S = \Sigma\, y_j x_j$ is sufficient

```
postscript(file="Po.ps",hor=F)
par(mfrow=c(3,1))
n<-c(200,200,200,200,200)
r<-c(32,51,70,103,148)
y<-r/n
x<-c(5,10,15,20,30)
plot(x,y,main="Coupon effectiveness study - proportion redeemed",xlab="price reduction (cents)
",ylab="proportion redeemed",las=1,xlim=c(0,40),cex=1.,ylim=c(0,1))
junk1<-lm(y~x)
xaxis<-c(0:40)
ylin<-junk1$coef[1]+junk1$coef[2]*xaxis
lines(xaxis,ylin,lty=3)
plot(x,log(y/(1-y)),main="Logit plot",xlab="reduction",ylab="logit(p)",las=1,xlim=c(0,40),cex=
1.)
junk<-glm(y~x,wei=n,family=binomial)
summary(junk)
anova(junk)
xaxis<-c(0:40)
ylin<-junk$coef[1]+junk$coef[2]*xaxis
ypred<-exp(ylin)/(1+exp(ylin))
plot(x,y,main="Fitted function",xlab="price reduction (cents)",ylab="proportion redeemed",las=
1,xlim=c(0,40),ylim=c(0,1),cex=1.)
lines(xaxis,ypred)
junk2<-residuals(junk,type="pearson")
X2<-sum(junk2^2)
pval<-1-pchisq(X2,3)
print(c(X2,pval))
graphics.off()
q()
```

Effectiveness of coupons offering a price reduction

1000 homes selected

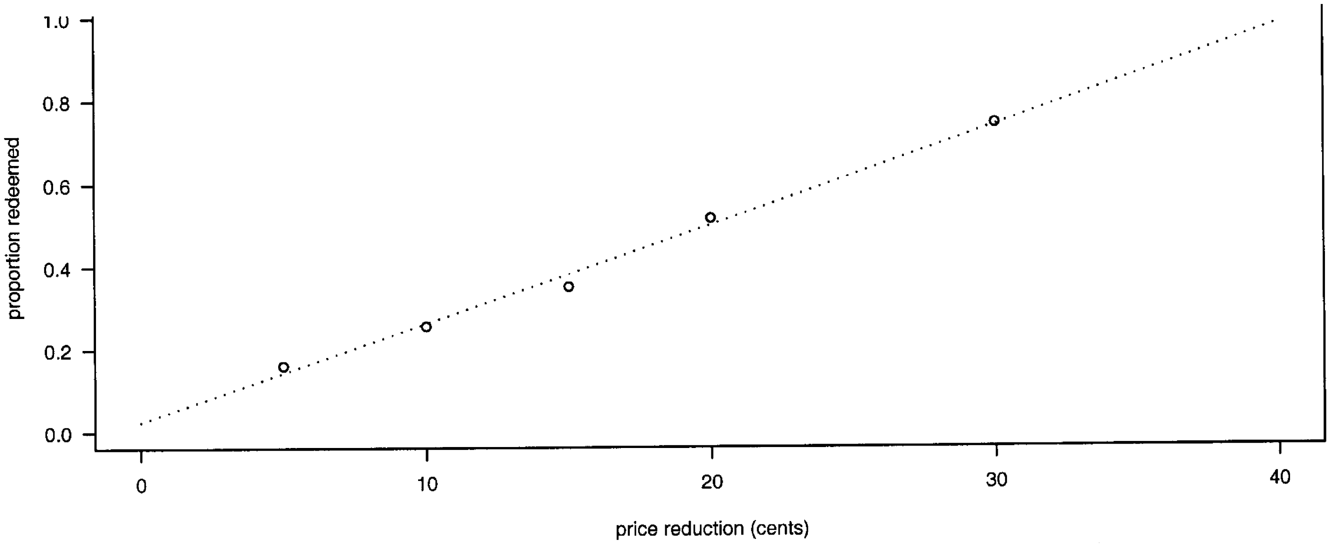
coupon and advertising mailed

price reductions of 5, 10, 15, 20, 30

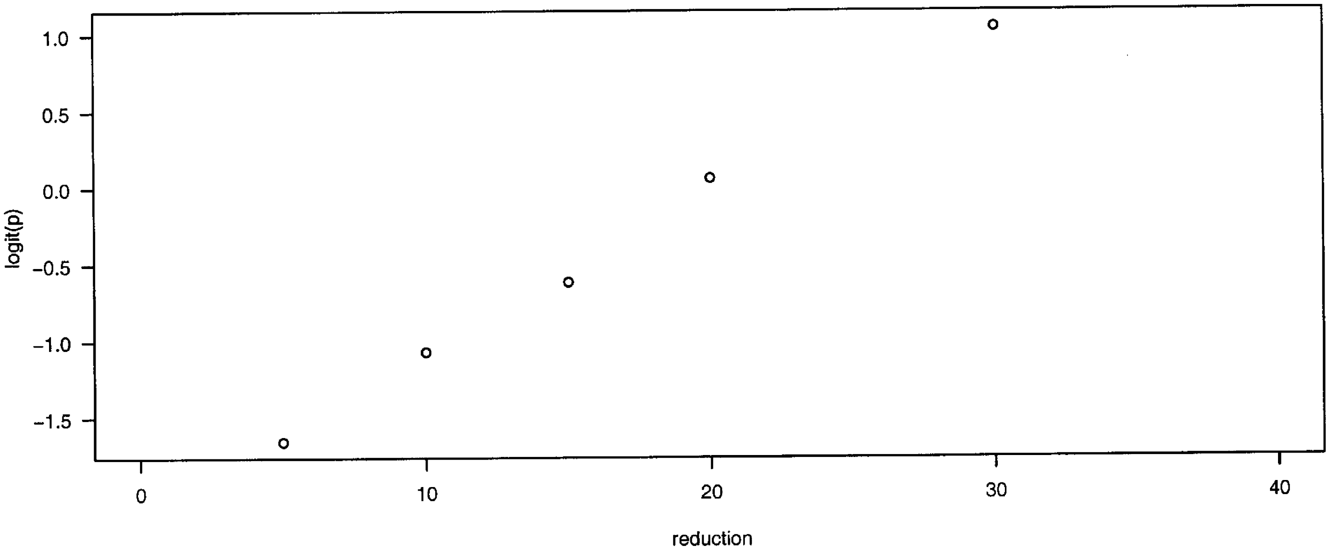
200 homes assigned at random to each price reduction, X

Y = 1 coupon redeemed within 6 months

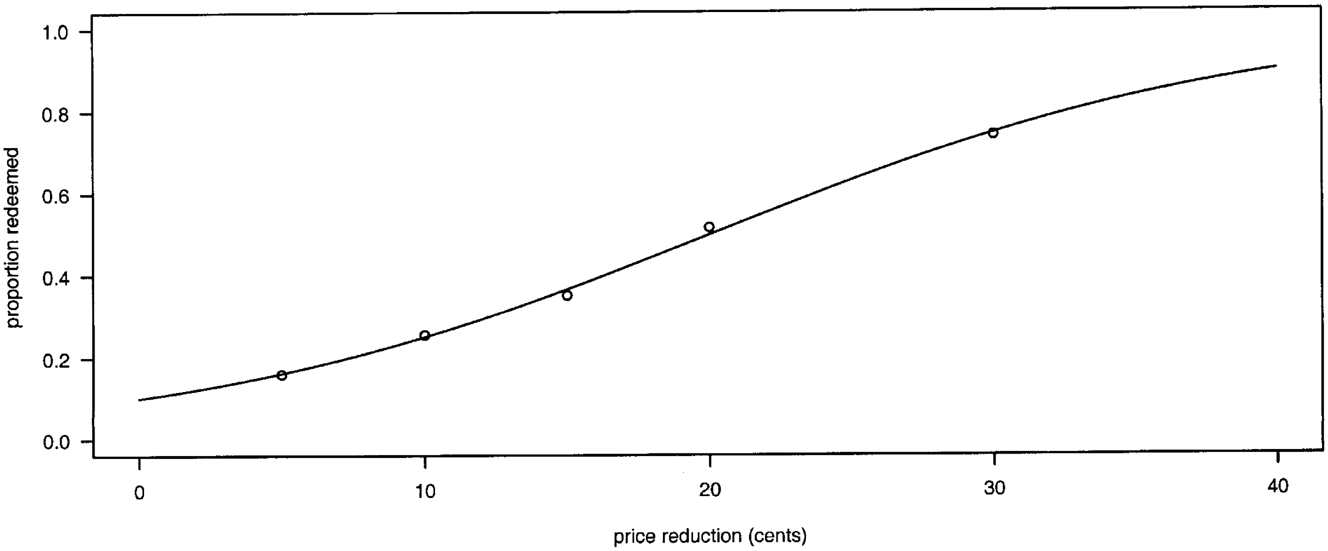
0 not



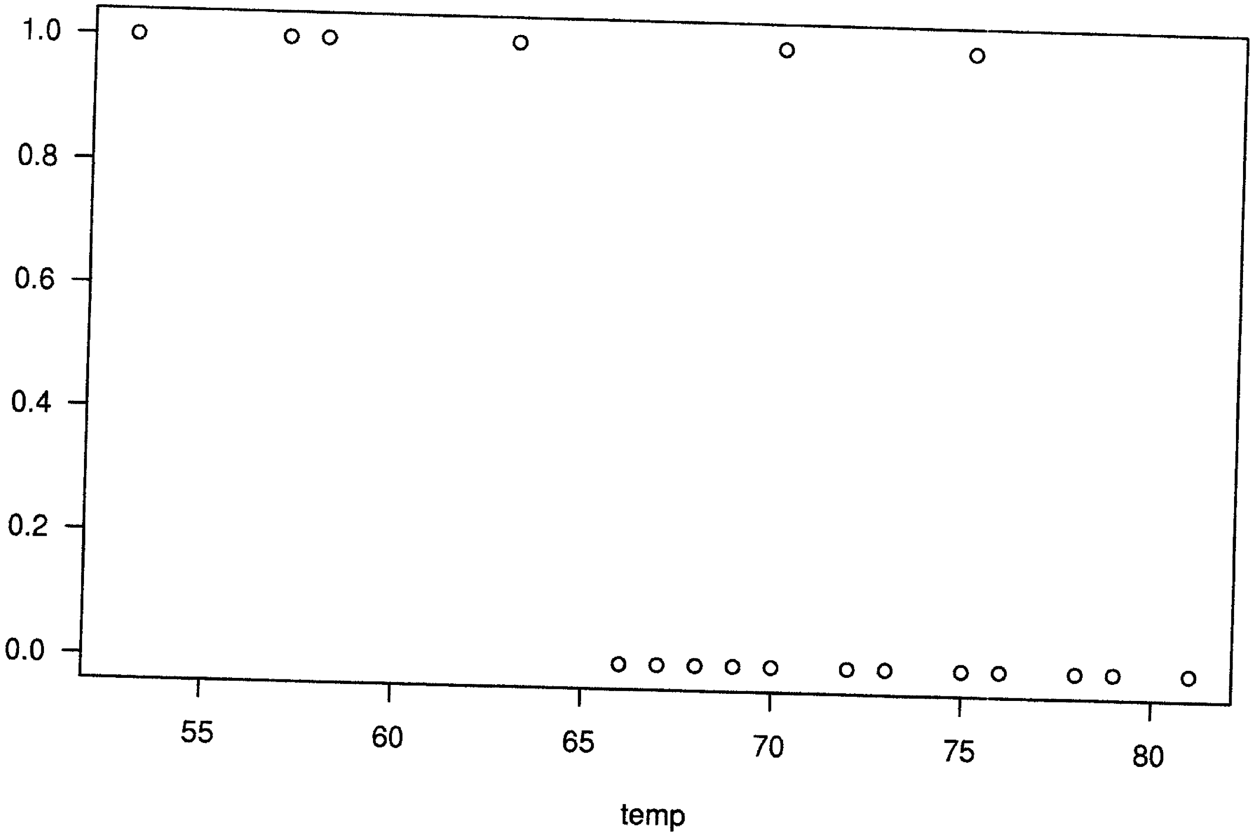
Logit plot



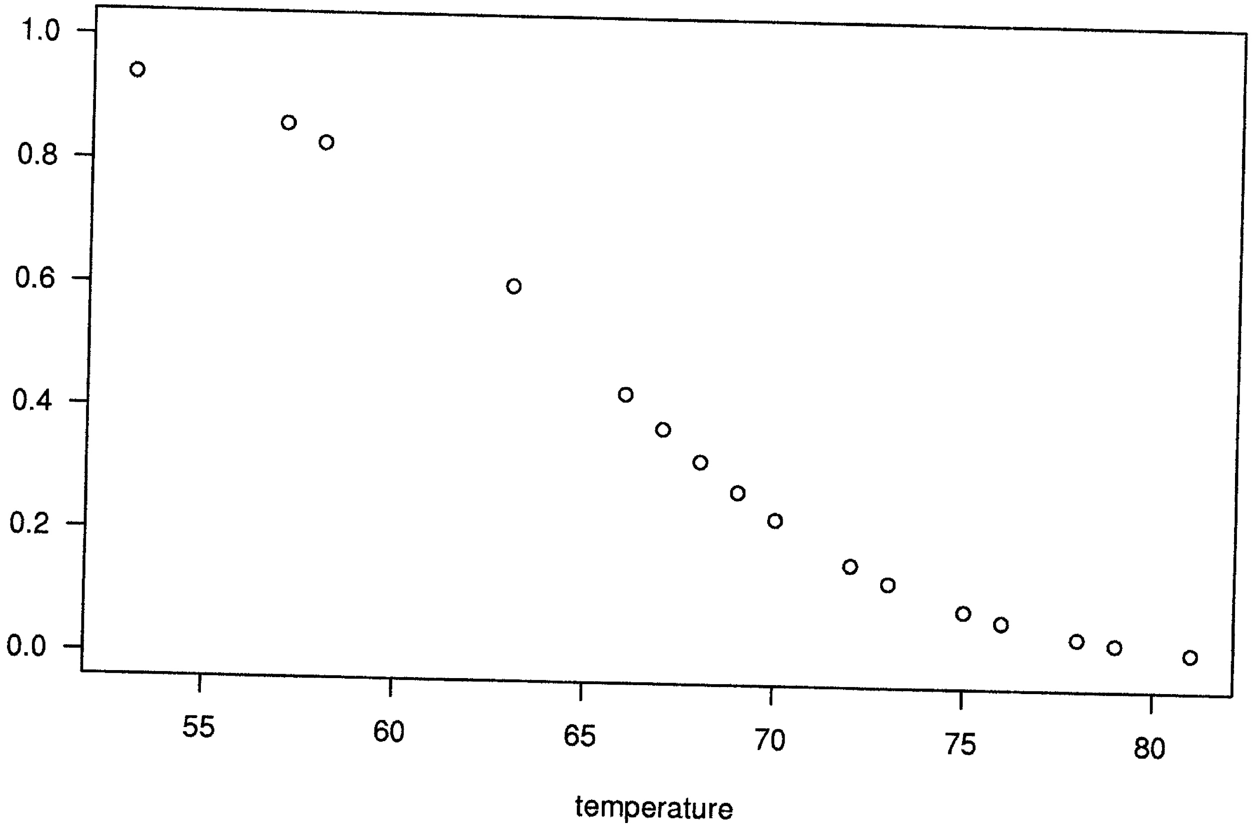
Fitted function

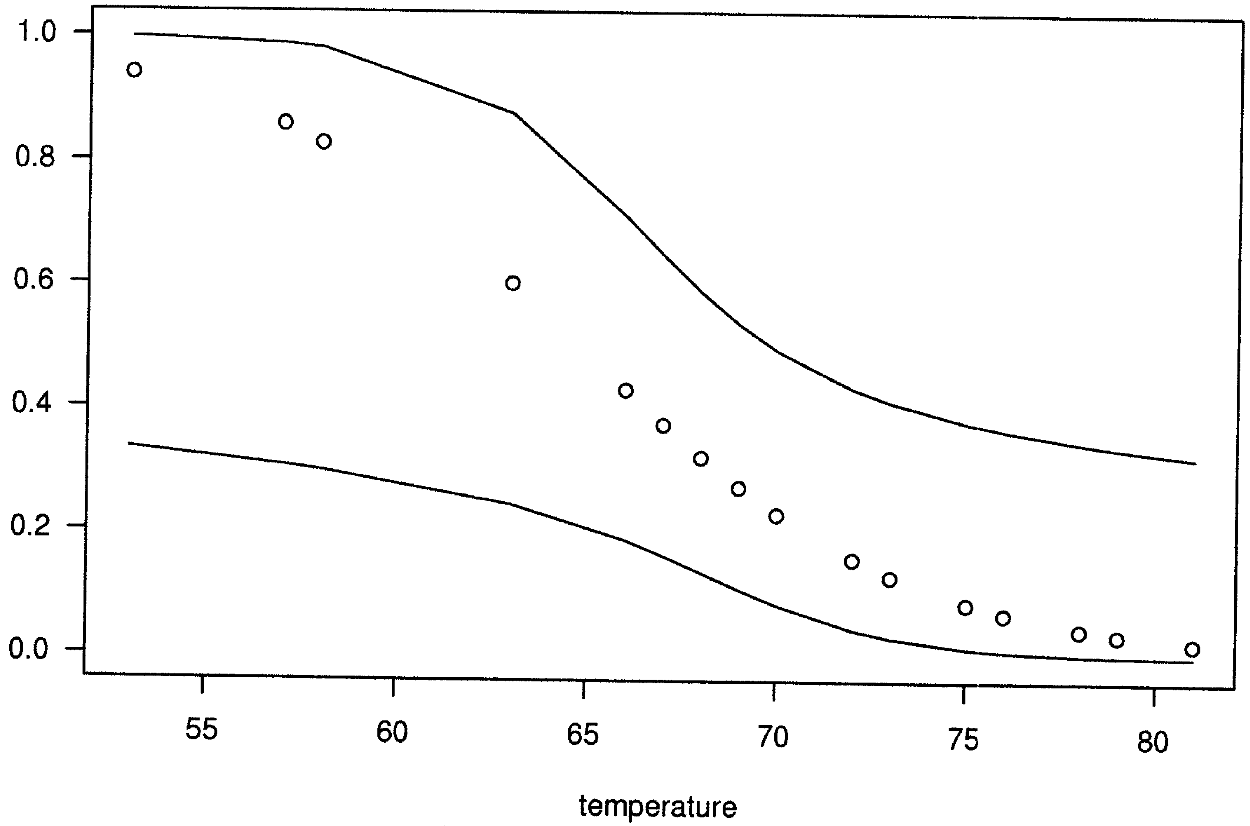


```
Call:
glm(formula = y ~ x, family = binomial, weights = n)

Deviance Residuals:
       1         2         3         4         5
-0.08476   0.16111  -0.43523   0.50294  -0.18626

Coefficients:
            Estimate Std. Error z value Pr(>|z|)
(Intercept) -2.185505   0.164667  -13.27   <2e-16 ***
x            0.108719   0.008843   12.29   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 180.94751  on 4  degrees of freedom
Residual deviance:   0.51021  on 3  degrees of freedom
AIC: 32.025

Number of Fisher Scoring iterations: 3

> anova(junk)
Analysis of Deviance Table

Model: binomial, link: logit

Response: y

Terms added sequentially (first to last)


     Df Deviance Resid. Df Resid. Dev
NULL                     4     180.95
x     1   180.44         3       0.51
[1] 0.5093288 0.9168372
```

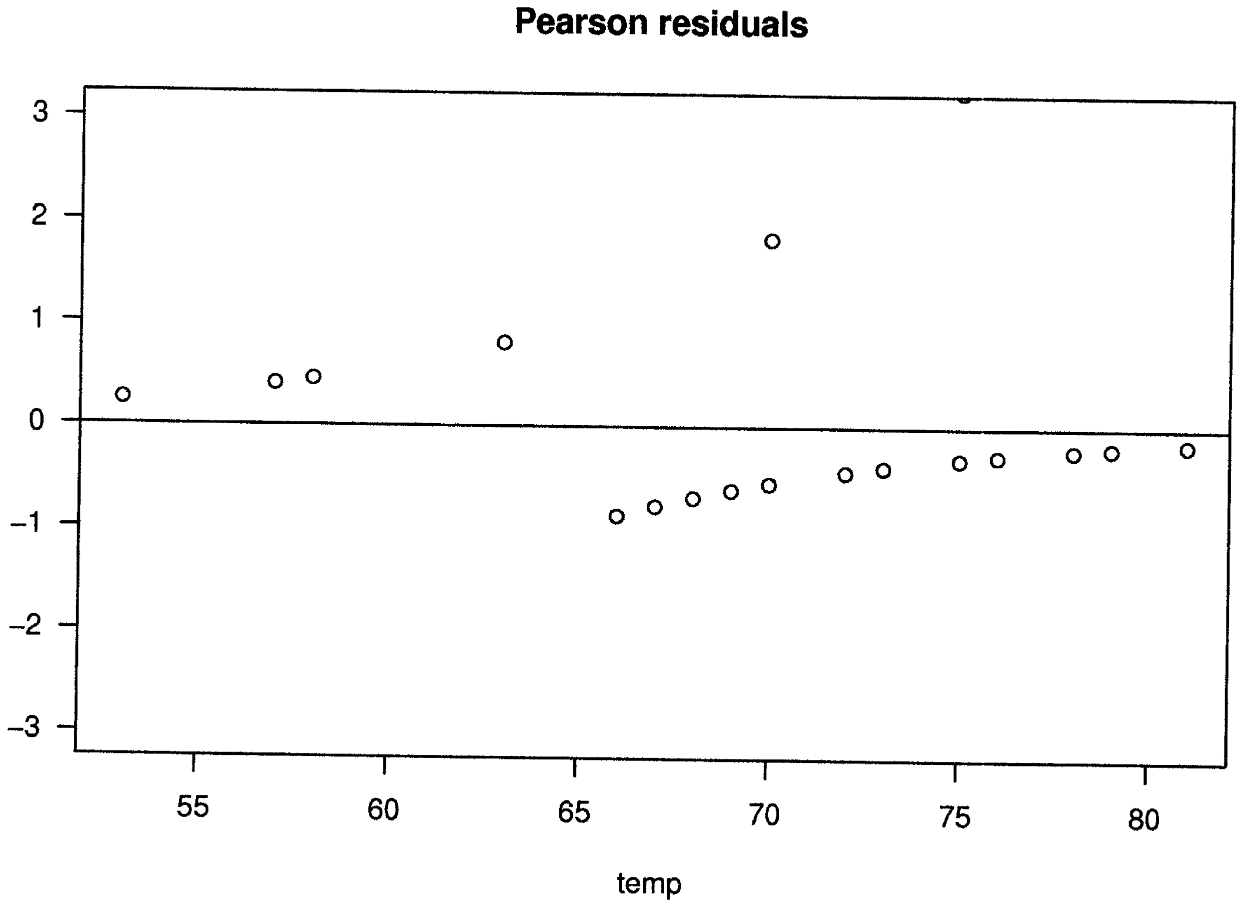


Fitted probabilities





95% CI for risk at 31 degrees F is (.441,1.00)



Model Checking

Residuals

deviance residual

$$d_j = \text{sign}(\tilde{\eta}_j - \hat{\eta}_j)\left[2\{\ell_j(\tilde{\eta}_j;\phi) - \ell_j(\hat{\eta}_j;\phi)\}\right]^{1/2}$$

Pearson residual

$$\frac{1}{\sqrt{\phi}}\,\frac{y_j - \hat{\mu}_j}{V(\hat{\mu}_j)/a_j}$$

Terms	df	Deviance reduction	Terms	df	Deviance reduction
Species (unadj. for Colour)	5	19.64	Species (adj. for Colour)	5	41.18
Colour (adj. for Species)	7	47.31	Colour (unadj. for Species)	7	25.78
Species (unadj. for Colour)	4	27.63	Species (adj. for Colour)	4	35.18
Colour (adj. for Species)	7	18.03	Colour (unadj. for Species)	7	10.48

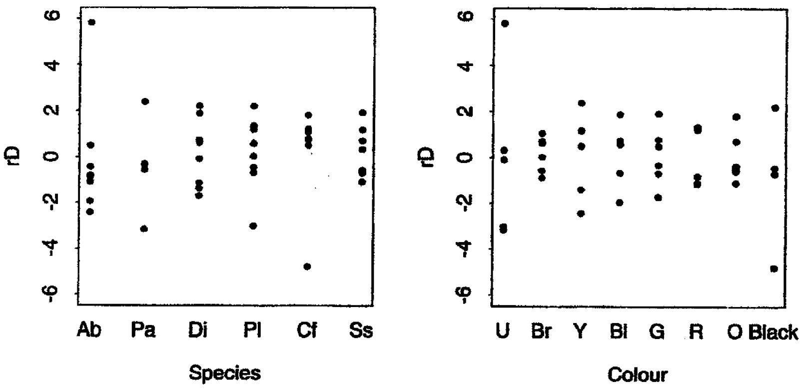


Figure 10.5 Standardized devian residuals r_D for bino two-way layout fitted jacamar data.

2 X 2 table

patients
- treatment or control
- success or failure

		Success	Failure	Total
m_1	Treated	R_1	$m_1 - R_1$	m_1
m_0	Control	R_0	$m_0 - R_0$	m_0
	Total	$R_1 + R_0$	$m_1 + m_0 - R_1 - R_0$	$m_1 + m_0$

Treatments assigned randomly

One model

$$\pi_1 = \frac{e^{\lambda+\psi}}{1+e^{\lambda+\psi}}, \quad \pi_0 = \frac{e^{\lambda}}{1+e^{\lambda}}$$

$$\frac{\pi}{1-\pi_1} = e^{\lambda+\psi} \qquad \frac{\pi_0}{1-\pi_0} = e^{\lambda}$$

$$\log \frac{\pi_1}{1-\pi_1} \Big/ \frac{\pi_0}{1-\pi_0} = \psi$$