Some notations and formulas - Chapter 4

\( \gamma(h) = \text{Cov}(X_{t+h}, X_t), \ h = 0, \pm 1, \pm 2, \ldots \) \hspace{1cm} (1)

\[ \sum_{h=-\infty}^{\infty} |\gamma(h)| < \infty \] \hspace{1cm} (2)

**Spectral density at frequency \( \lambda \)**

\( f(\lambda) = \frac{1}{2\pi} \sum_{h=-\infty}^{\infty} e^{-i\lambda h} \gamma(h), \ -\infty < \lambda < \infty \)

\[ = \frac{1}{2\pi} \left( \gamma(0) + 2 \sum_{h=1}^{\infty} \cos(\lambda h) \gamma(h) \right) \] \hspace{1cm} (3)

**Periodogram**

\[ I_N(\lambda) = \frac{1}{N} \sum_{t=1}^{N} x_t e^{-it\lambda} \] \hspace{1cm} (4)

\[ f_N(\lambda) = \frac{1}{2\pi} E(I_N(\lambda)) \] \hspace{1cm} (5)

Suppose \( \{X_t\} \) has mean 0, then \( I_N(\lambda) \) is asymptotically unbiased

\[ \lim_{N \to \infty} f_N(\lambda) = \lim_{N \to \infty} E(I_N(\lambda)) = f(\lambda) \] \hspace{1cm} (6)

Some properties

\( f(-\lambda) = f(\lambda) \) \hspace{1cm} (7)

\( f(\lambda) \geq 0 \) \hspace{1cm} (8)

\[ \gamma(k) = \int_{-\pi}^{\pi} e^{ik\lambda} f(\lambda) d\lambda = \int_{-\pi}^{\pi} \cos(k\lambda) f(\lambda) d\lambda, \ k = 0, \pm 1, \pm 2, \ldots \] \hspace{1cm} (9)

\[ f(\lambda + 2\pi) = f(\lambda) \] \hspace{1cm} (10)

Just as there are continuous and discrete random variables there are continuous and discrete spectra.