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JOHN WILDER TUKEY  
1915—2000

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*A Biographical Memoir by*  
DAVID R. BRILLINGER

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*Biographical Memoir*

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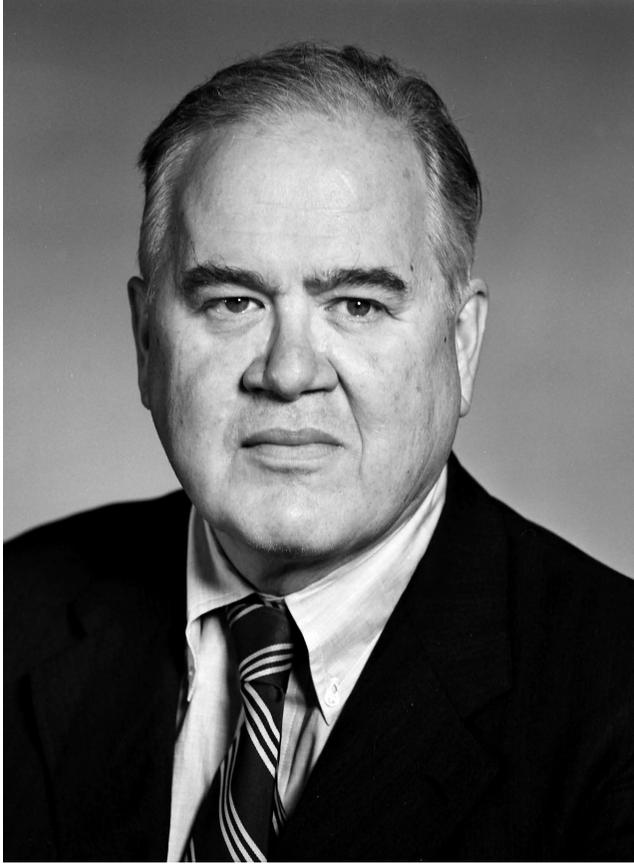


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*John W. Tukey*

# JOHN WILDER TUKEY

*June 16, 1915–July 26, 2000*

BY DAVID R. BRILLINGER<sup>1</sup>

JOHN WILDER TUKEY WAS renowned for research and service in each of academia, industry, and government. He was born June 16, 1915, in New Bedford, Massachusetts, the only child of Adah M. Tasker and Ralph H. Tukey. His parents had graduated first and second in the Bates College class of 1898. John had unusual talents from an early age. He could read when he was three, and had remarkable powers of mental calculation. His father had obtained a doctorate in Latin from Yale University and then moved on to teach and be principal at New Bedford High School. His mother was a substitute teacher there. Perhaps in consequence of these backgrounds, Tukey was schooled at home, but he did attend various classes at the high school.

Tukey's wife, Elizabeth Rapp, was born on March 2, 1920, in Ocean City, New Jersey. She went to Temple University and later was valedictorian in the 1944 class in business administration at Radcliffe College. About meeting John, she commented to the effect that the first time she saw him he was sitting in the front row of a public lecture asking quite difficult questions of the speaker. They later met at a folkdance class that John was teaching. Elizabeth and John married on July 19, 1950. Quoting Elizabeth, they were a "team." The Tukeys spent time at their old sea captain's

house in Westport Point, Massachusetts, and their house in Princeton was filled with American antique furniture.

The Tukeys had no children, but they did have a large extended family of blood relatives, ex-students, colleagues, and others. The statistician Francis J. Anscombe was his “brother-in-squared-law.” John and Francis (“Frank”) married the Rapp sisters, Elizabeth and Phyllis. After Elizabeth died on January 6, 1998, John remarked poignantly that “one is so much less than two.”

Tukey’s contributions to education and science continued right up until his death. That occurred after a brief illness and following a heart attack on July 26, 2000. The place of death was New Brunswick, New Jersey. I had spoken with John about a week earlier. His immediate question was whether I had a problem for him to think about.

John’s books have gone to Brown University and his papers to the American Philosophical Society, but the stories and gratitude remain in people’s minds and hearts.

#### CAREER

Tukey entered Brown University in 1933 by way of College Board Exams. There he earned bachelor’s and master’s degrees in chemistry in 1936 and 1937, respectively. Brown university was important to him during his whole life. He spoke about it often and wore the Brown necktie in formal situations. He was a member of their Board of Fellows from 1974 to 1988. During other time periods, he was a member of their Committee on Computers in Education, the Library Committee, and the Consultation Committee. He was also occasionally involved in Mathematics Department personnel affairs.

For a doctorate Tukey went to Princeton. Soon after arrival he switched from studying chemistry to studying mathematics. He had learned that the professor of chemistry was away

on leave for a period. John lived at the Graduate College. There and at Fine Hall he came to know and interact with Richard Feynman, Alan Turing, Oscar Morgenstern, Marston Morse, and John von Neumann, among others. He obtained his Ph.D. in 1939 after submitting a thesis titled “On Denumerability in Topology.”

John’s name appears in various of the books by and about Feynman. One special story involving the two relates to the keeping of time. Feynman knew that he himself could keep track of time while reading but not while speaking. He presented doing the latter as a challenge. Rising to it, Tukey showed that he had that other ability. Of what had happened Feynman remarked, “Tukey and I discovered that what goes on in different people’s heads when they think they’re doing the same thing—something as simple as counting—is different for different people.” This may be one source of Tukey’s oft made remark, “People are different.”

Tukey’s supervisor was Solomon Lefschetz and thus his scientific ancestors included two Bernoullis, Euler, Lagrange, Fourier, Poisson, Gauss, Dirichlet, Jacobi, Lipschitz, Klein, and Neumann (Mathematics Genealogy Project, 2009). His Erdos number is 2.

Tukey remained at and in Princeton his whole career. He was an instructor at the university from 1939 to 1941, and then an assistant professor from 1941 to 1948. With the advent of the war in Europe, Tukey came to be a research associate of the Fire Control Research Office in Princeton. There he worked on problems of stereoscopic rangefinders, B-29 gunfire control, and computing leads for machine guns aimed at fighter planes. He was also involved in the Enigma code-breaking operations. During that period, Tukey moved into problems whose solutions involved statistics and data analysis. In later years he commented often on the influences that Edgar Anderson and Charles P. Winsor had on his de-

velopment during that period. He also acknowledged that he had learned statistics by reading the papers and discussions in the *Journal of the Royal Statistical Society Supplement*.

When the war ended, Tukey took up joint appointments in mathematics at Princeton and at Bell Telephone Laboratories. This job split continued until Tukey's retirement. At Princeton he became an associate professor in 1948 and professor in 1950. In 1966 he became the first chair of the Statistics Department. At the labs he worked his way up the managerial ladder ending with the position of associate executive director of information sciences. In the late 1950s Tukey was instrumental in setting up the Statistical Techniques Research Group and also an Institute for Defense Analyses branch in Princeton.

Throughout his career Tukey was called upon for his advice and ideas as a consultant. In 1954 he was a member of the Killam Committee, a group charged by President Eisenhower to study the possibility of a surprise nuclear attack. That committee recommended the construction of the U-2 airplane. In 1959 he was one of the U.S. delegates to Technical Working Group 2 at the Conference on the Discontinuance of Nuclear Weapon Tests, in Geneva. John's expertise in part concerned the scientific problem of distinguishing earthquakes from nuclear explosions. To address this problem Tukey sought improved estimates of the directions of first motion of the seismic vibrations surrounding the event. This was motivated by the fact that the theoretical pattern of first motions about an event differed between explosions and earthquakes. He also developed a novel procedure, cepstral analysis, to estimate the depth of the event, having in mind that the depths of explosions were not nearly as great as those of earthquakes.

Tukey was a member of the President's Scientific Advisory Committee for a period starting in 1960. All told he advised

five U.S. presidents. From 1965 until his death he was a consultant to the Educational Testing Service. He also had a long involvement with the National Assessment of Educational Progress, based first in Denver and later at ETS in Princeton. During 1970-1971, Tukey was a member of the President's Commission on Federal Statistics. In 1972 he was a member of the U.S. delegation to the United Nations Conference on the Human Environment, in Stockholm. Tukey advised the Census Bureau for many years. In 1980 the Census Bureau's step of adjusting the raw values that had been collected to obtain "improved" estimates, became a hot political issue. Tukey testified in support of adjustment in the court case that resulted. Other consultancies were with Merck & Company, Radio Corporation of America, Bellcore, Xerox Palo Alto Research Center, and the Health Effects Institute.

He was an officer in a wide variety of professional societies: American Meteorological Society Council, vice president, American Statistical Association; vice president, Society of Industrial and Applied Mathematics; and president, Institute of Mathematical Statistics. He was a member of many National Academy of Sciences committees. Tukey retired from Princeton and Bell Telephone Laboratories in 1985, but the consulting and statistical research continued until his death.

#### TECHNICAL CONTRIBUTIONS

Tukey made contributions to each of engineering, mathematics, statistics, and science. Many of these are recorded in *The Collected Works* whose volumes began to appear in 1984. The editor in chief of the Works was William S. Cleveland. To illustrate the range of topics note that the volumes have the titles: *Time Series* (2 vol.); *Philosophy and Principles of Data Analysis* (2 vol.); *Graphics; More Mathematical; Factorial and ANOVA*; and *Multiple Comparisons*. They were handled by vari-

ous editors. In each volume an editor discusses the papers included, and there is a reply by Tukey. The last volume of the Works appeared in 1994.

#### MATHEMATICS

The *More Mathematical* volume includes a section on “Mathematics,” which contains papers on linear functionals, compactness in general spaces, separation of convex sets, generalized “sandwich” theorems, and linearization of solutions in supersonic flow.

Tukey’s reputation in mathematics became serious when he reached Princeton, and that reputation remains to the present. As a graduate student and later Tukey attended many classes and lectures; throughout his career he had a reputation for dominating their proceedings. Of lectures by Marston Morse on symbolic dynamics, Tukey remarked that Steenrod and he did their best to keep “Morse honest.” Keeping people honest remained a key part of Tukey’s scientific life. In a course that Albert Tucker presented in 1938, Tucker once remarked that every time he gave a definition of a combinatorial manifold Tukey would come up with a counterexample. Tucker summarized the interchanges as ending in a draw.

Parts of Tukey’s doctoral thesis appeared in the first *Annals of Mathematics Studies* volume (1940). That book is still referenced today and remains available from the Princeton University Press. In it Tukey has an approach to uniform spaces different from those of A. Weil and N. Bourbaki. Later Tukey’s approach was used extensively by J. R. Isbell and N. R. Howse in their books. Howse wrote, “Tukey’s contribution in this area went beyond showing us how we should think about uniform spaces. His insight was almost prophetic...He predicted that normal uniform spaces would play a major role in mathematics, and indeed they have.”

Tukey had papers with R. P. Boas and with A. Stone. The one with Stone showed that the volumes of any  $n$  solids in  $R^n$  can be simultaneously bisected by an  $(n - 1)$  sphere, with a plane able to be regarded as a sphere of infinite radius. Tukey also did some recreational mathematics in the late 1930s, in particular, concerning the theory of flexagons. He worked on this topic with R. Feynman, A. Stone, and B. Tuckerman. An Internet search shows that Tukey's name lives on, besides concerning flexagons, in mathematics more generally. One turns up Galois-Tukey connections, Tukey equivalence, Tukey reducibility, Tukey theory of analytic ideals, Tukey ordering, and of course, Tukey's lemma.

#### ENGINEERING

Tukey's work at Bell Telephone Laboratories has already been mentioned. As World War II was ending he designed the electronic adding circuit to be used in the Institute for Advanced Studies von Neumann computer. Tukey's name is on a number of related patents. As mentioned above in the 1950s, he worked on the design of the U-2 airplane. Through his work at Bell Labs Tukey got interested in the frequency analysis of time series. Hamming and he introduced a viable family of estimates of the spectral density of a time series, and Tukey developed their statistical properties—thus leading to improved estimates. He introduced the related method of complex demodulation that has proved so useful for the frequency analysis of nonstationary time series.

The discovery that brought Tukey's name to worldwide prominence was the Fast Fourier Transform (FFT). In 1963 in a graduate course at Princeton he showed that if one wished to evaluate the discrete Fourier transform of a stretch of  $N$  numbers at the Fourier frequencies, then when  $N$  factorizes into  $GH$  the computation of the empirical Fourier transform required but  $(H + 2 + G)GH$  multiplications. He

further remarked that for  $N = 4^k$  one needed fewer than  $2N + N \log_2 N$ . His lectures (Tukey, 1963) are reproduced in the *Time Series* volume of *The Collected Works*. The renowned Cooley and Tukey paper (1965) resulted in and has become a citation classic. The impact of the work was astonishing. Almost overnight engineers switched from analog to digital signal processing in countless cases.

Another aspect of digital signal processing that Tukey had picked up on was the occurrence of Gibbs phenomenon. He studied the effects of employing convergence factors to reduce the “leakage” effect. He wrote concerning the difficulties of interpretation of the results of a spectrum analysis, for example, the aliasing of frequencies caused by equispaced sampling of a series. The book by Blackman and Tukey (1958) was highly influential in introducing researchers to numerical spectrum analysis.

Turning to another engineering topic, Tukey was an important contributor in the field of acceptance sampling. This field had long been of importance at Bell Labs. He received two medals, the Deming and the Shewhart, from the American Society for Quality Control.

#### STATISTICS

Tukey contributed to the statistical topics of: algorithms, analysis of variance, confirmatory data analysis, exploratory data analysis, environmetrics, expert systems, fiducial probability, graphics, military analysis, multiple comparisons, medical statistics, Monte Carlo, multivariate analysis, nonparametrics, path coefficients, propagation of error, psephology, randomization, regression, robustness, sampling, smoothing, statistical philosophy, statistical practice, statistical theory, statistical education, and time series, among others. In fact the *Encyclopedia of Statistical Science* contains 233 entries mentioning Tukey.

*Sampling.* In 1950 following the substantial criticism of A. C. Kinsey's book, *Sexual Behavior in the Human Male*, the American Statistical Association assembled a committee to review the statistical problems arising. The committee members were W. G. Cochran, F. Mosteller, and J. W. Tukey. They were concerned particularly with the sampling methods and the absence of controlled randomness in Kinsey's work. The committee's efforts led to substantial advances in the theory and understanding of sampling methods.

Tukey also developed the sampling properties of certain polynomials in data values, the polykays, which are generalizations of the traditional sample mean and variance and particularly useful when the data are not close to normal. He further introduced the concept of configural polysampling in his von Neumann lecture.

*Robust methods.* Tukey became interested in the problems of robustness and resistance in the last stages of his work at the FRCO. These topics refer to procedures that are insensitive to non-normality and to the presence of outlying values, respectively. For example the scatter of some of the FRCO measurements appeared to be longer tailed than the normal. Robust methods became a continuing interest and, for example, led him to the consideration of the properties of estimates such as trimmed means and biweights. Kafadar (2002) provides a review.

*Exploratory data analysis and confirmatory data analysis.* Tukey developed in parallel the fields of exploratory and confirmatory data analysis discovering novel methods for each. After the FFT's (re)discovery, Tukey is probably most widely known for reenergizing the field of descriptive statistics. It is concerned with laying bare surprises in the data, suggesting hypotheses and for highlighting departures from assumptions. His paper, Tukey (1962), changed the language and paradigm of contemporary statistics. Tukey discovered and named the

jackknife procedure meant to provide a general estimate of variance. It evaluates a broadly computable indication of the uncertainty of an estimate by judiciously combining estimates based on subsets of the full dataset. He was also a strong proponent of the use of randomization distributions in obtaining p-values and confidence intervals.

*Statistical graphics.* Many of Tukey's graphical techniques were developed for exploratory data analysis. His boxplots and stem-and-leaf diagrams now appear throughout scientific presentations and high school texts. His dynamic graphics methods, such as PRIM-9, allow examination of moderate dimensional multivariate datasets in a search for both regularities and irregularities.

*ANOVA (analysis of variance) and regression.* The methods of these fields are the workhorses of statistics. Tukey made many important contributions to each, partly through an emphasis on the consideration of residuals. An assumption of additivity is basic to many of the developments. Tukey was concerned with how to examine that assumption in practice and developed the one degree of freedom for nonadditivity statistics. Further, he contributed to fractional replication and the study of components of variance.

*Multiple comparisons.* Tukey struggled with the problem of how to control the probabilistic error rate when many statements are being made. Difficulties arise if one carries out several analyses of the same dataset. In the late 1940s Tukey proposed a method based on the studentized range. This is popularly known as Tukey's honestly significant difference (HSD) method.

*Multivariate analysis.* Among the many original ideas and techniques that Tukey proposed are Tukey depth, Tukey median, projection pursuit, dyadic ANOVA, and the bagplot.

*Information sciences.* Starting in the mid-1960s, John Tukey sought to bring order to the literature of statistics and prob-

ability by constructing indices of the papers of those fields. He had done extensive work previously for *Mathematical Reviews* and prepared bibliographies (e.g., for time series). In particular, he constructed a statistical citation index, the first outside the legal profession.

#### CONCLUDING REMARKS

John Tukey was a professor and an executive. He merged the scientific, governmental, technological, and industrial worlds more seamlessly than perhaps anyone else in the 20th century. His scientific knowledge, creativity, experience, calculating skills, and energy were prodigious. He was renowned for conceptualizing problems, creating novel statistical concepts and coming up with both new words for new ideas and new uses for old words. There are few branches of statistics that his ideas have not impinged upon. He introduced algorithms, concepts, language, philosophy, and techniques. He made important contributions to science. He set down a prescription, headed “ $Rx$ ,” for handling any statistical problem that a scientist came to him with. He had many coauthors. He was one of the great statistical scientists of the 20th century.

Tukey’s strengths were the science he knew, the people he interacted with, the energy he had, his intuition, and his creativity. He controlled discussions and lectures. He was speculative. He once remarked that the best way to get the issues onto the table was to start an argument, but sometimes avoided them, for example, those over Bayesian statistics. He liked to work with people that he got along with.

Tukey had scientific impact both by writings and in person, the latter perhaps being the most important. He had many students and seemed to enjoy teaching a great deal. An important part of his students’ education was the Princeton Applied Statistics Seminar. The speakers were told that they did not

need to have a solution to their problem. The participants, including particularly the students, were encouraged to speak up even when unsure of what they had to say.

There are interviews both on paper, for example, Fernholz and Morgenthaler (1997), and in videos. There are memorial articles in numbers of *Annals of Statistics*, *Statistical Science*, and *Technometrics*. Many contain anecdotes. Tukey wrote and edited many books, often with D. Hoaglen and F. Mosteller.

Tukey was a member of various committees of the National Academy of Sciences. His nephew F. R. Anscombe in a presentation at the library of the American Philosophical Society in 2007, speculated on various other involvements that Tukey might have had. Time may confirm them.

Tukey was a kind and gentle person. When he telephoned, having learned that my son had died after a long struggle with cancer, he was crying. A great man has moved on but has left footprints in many places.

#### MAJOR HONORS

The level and breadth of Tukey's career is made apparent by the following major honors:

- 1949 Guggenheim fellow
- 1958 Wald lecturer, Institute of Mathematical Statistics
- 1961 Member, National Academy of Sciences
- 1962 Member, American Philosophical Society
- 1964 Member, American Academy of Arts and Sciences
- 1965 S. S. Wilks Medal, American Statistical Association, for statistical contributions to the advancement of scientific or technical knowledge, ingenious application of existing knowledge, or successful activity in the fostering of cooperative scientific efforts that have been directly involved in matters of national defense or public interest.

- 1967 Fisher lecturer, Committee of President of Statistical Societies
- 1973 The National Medal of Science for his studies in mathematical and theoretical statistics, particularly his pioneering work on broad analysis and synthesis problems of complex systems, and for his outstanding contributions to the applications of statistics to the physical, social, and engineering sciences
- 1975 Hitchcock lecturer, University of California, Berkeley
- 1975 Scott lecturer, Cambridge University
- 1976 Shewhart Medal, American Society for Quality Control, for outstanding technical leadership in the field of modern quality control
- 1982 Medal of Honor, Institute of Electrical and Electronic Engineers, for his contributions to the spectral analysis of random processes and the fast Fourier transform algorithm
- 1983 Deming Medal, American Society for Quality Control, for successfully combining the application of statistical thinking and management so that each supports and enhances the other, thus leading to quality in products and services
- 1984 James Madison Medal, Princeton University, presented each year to an alumnus or alumna of the Graduate School who has had a distinguished career, advanced the cause of graduate education, or achieved an outstanding record of public service
- 1985 John von Neumann lecturer, Society of Industrial and Applied Mathematics
- 1986 Honorary fellow, Royal Statistical Society
- 1989 Monie A. Ferst Award, Sigma Xi, for science and engineering teachers who have inspired their students to significant research achievements
- 1990 Educational Testing Service Award for Distinguished Service to Measurement
- 1991 Foreign member, Royal Society of London

There were honorary doctorates from the following universities: Case Institute, Brown, Yale, Chicago, Temple, Princeton, and Waterloo.

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