Stat153 Assignment 2 (due September 24, 2010)

1. **(ACF of MA)**

   (a) Find the autocovariance function of the time series $X_t = W_t + \frac{5}{2}W_{t-1} - \frac{3}{2}W_{t-2}$, where $\{W_t\} \sim WN(0,1)$.

   (b) Find the autocovariance function of the time series $X_t = \tilde{W}_t - \frac{1}{6}\tilde{W}_{t-1} - \frac{1}{6}\tilde{W}_{t-2}$, where $\{\tilde{W}_t\} \sim WN(0,9)$. Compare with 1a.

   (c) Which of the MA models in 1a and 1b is invertible?

2. **(ARMA models)**
   For each of the following ARMA models, find the roots of the AR and MA polynomials, identify the values of p and q for which they are ARMA(p,q) (be careful of parameter redundancy), determine whether they are causal, and determine whether they are invertible. In each case, $\{W_t\} \sim WN(0,1)$.

   (a) $X_t + 0.81X_{t-2} = W_t + \frac{1}{3}W_{t-1}$.

   (b) $X_t - X_{t-1} = W_t - \frac{1}{2}W_{t-1} - \frac{1}{2}W_{t-2}$.

   (c) $X_t - 3X_{t-1} = W_t + 2W_{t-1} - 8W_{t-2}$.

   (d) $X_t - 2X_{t-1} + 2X_{t-2} = W_t - \frac{8}{9}W_{t-1}$.

   (e) $X_t - 4X_{t-2} = W_t - W_{t-1} + \frac{1}{2}W_{t-2}$.

   (f) $X_t - \frac{9}{4}X_{t-1} - \frac{9}{4}X_{t-2} = W_t$.

   (g) $X_t - \frac{9}{4}X_{t-1} - \frac{9}{4}X_{t-2} = W_t - 3W_{t-1} + \frac{1}{9}W_{t-2} - \frac{1}{3}W_{t-3}$. (Hint: This model is causal.)

3. **(linear process representation of ARMA)**
   For those models of Question 2 that are causal, compute the first five coefficients $\psi_0, \psi_1, \ldots, \psi_4$ in the causal linear process representation $X_t = \sum_{j=0}^{\infty} \psi_j W_{t-j}$.

4. **(ACF of ARMA)**
   For those models of Question 2 that are causal,

   (a) Compute the ACF.

   (b) Simulate 100 observations from each model. Compute and plot the sample ACF.