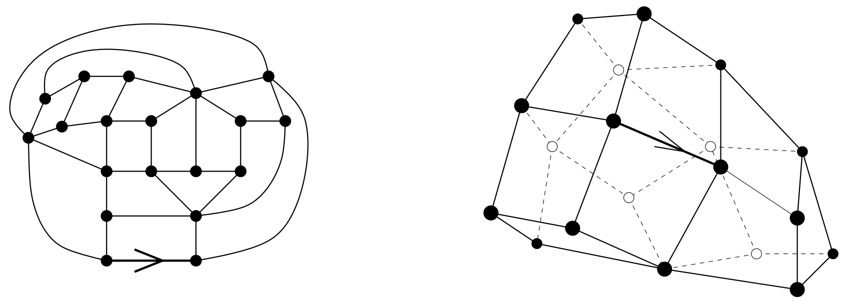

FIGURE 1. Random quadrangulations, in planar or spherical representation.

into baby universes [20]. In this article we consider another fundamental aspect of the geometry of random maps, namely *global properties of distances*. The *profile* $(H_k^n)_{k\geq 0}$ and *radius* r_n of a random quadrangulation with n faces are defined in analogy with the classical profile and height of trees: H_k^n is the number of vertices at distance k from a basepoint, while r_n is the maximal distance reached. The profile was studied (with triangulations instead of quadrangulations) by physicists Watabiki, Ambjørn et *al.* [4, 35] who gave a consistency argument proving that the only possible scaling for the profile is $k \sim n^{1/4}$, a property which reads in their terminology *the internal Hausdorff dimension is 4*. Independently the conjecture that $\mathbb{E}(r_n) \sim cn^{1/4}$ was proposed by Schaeffer [31].

Integrated SuperBrownian Excursion. On the other hand, ISE was introduced by Aldous as a model of random distributions of masses [1]. He considers random embedded discrete trees as obtained by the following two steps: first an abstract tree t, say a Cayley tree with n nodes, is taken from the uniform distribution and each edge of t is given length 1; then t is embedded in the regular lattice on $\mathbb{Z}^d$, with the root at the origin, and edges of the tree randomly mapped on edges of the lattice. Assigning masses to leaves of the tree t yield a random distribution of mass on $\mathbb{Z}^d$. Upon scaling the lattice to $n^{-1/4}\mathbb{Z}^d$, these random distributions of mass admit, for n going to infinity, a continuum limit $\mathcal{J}$ which is a random probability measure on $\mathbb{R}^d$ called ISE.

Derbez and Slade proved that ISE describes in dimension larger than eight the continuum limit of a model of lattice trees [15], while Hara and Slade obtained the same limit for the incipient infinite cluster in percolation in dimension larger than six [18]. As opposed to these works, we shall consider ISE in dimension one and our embedded discrete trees should be thought of as folded on a line. The support of ISE is then a random interval (L, R) of $\mathbb{R}$ that contains the origin.

From quadrangulations to ISE. The purpose of this paper is to draw a relation between, on the one hand, random quadrangulations, and, on the other hand, Aldous' ISE: upon proper scaling, the profile of a random quadrangulations is described in the limit by ISE translated to have support $(0, R - L)$. This relation implies in particular that the radius r_n of random quadrangulations, again upon scaling, weakly converges to the width of the support of ISE in one dimension, that is the continuous random variable $r = R - L$. We shall indeed prove (Corollary 3)