TABLE 1
*Some max-type RDEs. Functions $g(\cdot)$ for which the RDE $X \overset{d}{=} g((\xi_i, X_i), i \geq 1)$ are discussed**

Section	$g(\cdot)$	Underlying model	Endog?	Comments
	$S = \mathbb{R}^+$			
4.2	$\max_i (X_i + \xi_i)^+$	Range of BRW	Yes	
4.3	$\min_i (X_i + \xi_i)^+$	Algorithm for BRW range	Yes	
4.6	$\max_i (\xi_i - X_i)^+$	Matching on GW tree	Yes	
4.4	$\xi_0 + \max_i (\xi_i X_i)$	Discounted tree sums	Yes	$\xi_0 = 0$ reduces to BRW extremes
4.4	$\xi_0 + \min_i (\xi_i X_i)$	Discounted tree sums	Yes	See (49)
4.6	$(\xi_0 - \sum_i X_i)^+$	Independent subset GW tree	Yes	
7.2	$\sum_i (c - \xi_i + X_i)^+$	Percolation of MSTs	Yes	Determines critical c
7.6	See (98)	First passage percolation	Conj. Y	Mean-field scaling analysis
	$S = \mathbb{R}$			
5	$c + \max_i (X_i + \xi_i)$	Extremes in BRW	No	c specified by $\mathrm{dist}(\xi_i)$
7.3	$\min_i (\xi_i - X_i)$	Mean-field minimal matching	Yes	
7.4	$\min_i^{[2]} (\xi_i - X_i)$	Mean-field TSP	Conj. Y	$\min^{[2]}$ denotes second smallest
	Other S			
6	$\Phi(\min(X_1, X_2), \xi_0)$	Frozen percolation on tree	Yes	Φ defined in Section 6
7.6	See (96), (97), (98)	Mean-field scaling	Conj. Y	$S = \mathbb{R}^2$ or $\mathbb{R}^3$

*Note $x^+ = \max(x, 0)$. For $S = \mathbb{R}$ a "max" problem is equivalent to a "min" problem by transforming X to $-X$, but for $S = \mathbb{R}^+$ this does not work: the problems in Sections 4.2 and 4.3 are different. Typically the (ξ_i) are either i.i.d. or are the successive points of a Poisson process on $(0, \infty)$. "Endogenous" refers to fundamental solution. Key to acronyms: BRW, branching random walk; GW, Galton–Watson; MST, minimal spanning tree; TSP, traveling salesman problem.

1.1.1. *Direct use.* Here is the prototype example of *direct use*, where the original question asks about a random variable X and the distribution of X itself satisfies an RDE.

EXAMPLE 1. Let X be the total population in a Galton–Watson branching process where the number of offspring is distributed as ξ. In the case $\mathbb{E}\xi \leq 1$ [and $\mathbb{P}(\xi = 1) \neq 1$] it is well known that $X < \infty$ a.s., and then easy to check that $\mathrm{dist}(X)$ is the unique solution of the RDE

$$X \overset{d}{=} 1 + \sum_{i=1}^{\xi} X_i \qquad (S = \mathbb{Z}^+).$$

We will see other direct uses in Proposition 25 and in the examples in Section 4.4.

1.1.2. *Indirect use.* The simplest kind of indirect use is where the quantity of interest can be written in terms of known quantities and some other quantity