**Mathematical model for optimal cost-efficient installation of automatically operated public transportation system**

**(Introductory proposal paper)**

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Abstract

This thesis analyzes the way how theorems in network theory could be applied to choosing locations for placing each of bicycle stations in the city efficiently. Hence, it sets up a mathematical model for predicting paths and number of people interacting at the given time interval using graph theory and game theory. Then, it evaluates the cost efficiency of such system and concludes with case studies to show how the model could be used in real life setting.

(Assumption: all bicycles and maintenance facilities are of uniform fixed cost)

**Main hypothesis**

There exists an optimal installation of public transportation in any given place and we can model it with two different variables: time and distance.   
  
**Possible Applied Theorems**   
-Theorems about characteristics of directed graphs in Network theory: might be useful for drawing paths of people living in the city.

1. Directed graph : trace the moving paths of each individual at given time interval and form graph with **edges that represent the concurring paths of people** (set a lower bound and filter out unnecessary paths) and **vertices that represent the attraction place where people tend to gather and stay** ( not moving for certain time interval that is longer than the lower bound that we have set up)
2. Spanning matrix : after tracing the paths and attraction locations, we start to count the degree of each vertices and find draw nodes on the vertices with higher degrees. **These vertices then represent the most probable candidates since higher degree means that more people go through those points**.
3. Euclidian Trail and distance: due to the structure of city, which is usually compact with buildings, facilities, roads, and train rails, the graph is not complete. Then, we need to know in terms of not only physical distance but also **step-wise distance from given vertices A and B, each with degrees higher than 1.**
4. Small World Property : this is often used in network theory, and it is sometimes a very useful tool to use when **analyzing structure or mechanism of possible connections between many people.**

-Game theorems with time and distance as two possible variables: this might help us understand how individuals behave when it comes to choosing whether to ride a bicycle or not and the choice of bicycle ports.

**1.** Individualistic Game : this deals with the decision that **a single first perspective person with ability to think logically is making**. The variables are time and distance, and additionally, the choice of other people.   
 **2.** Behavior of people with fuzzy decision and lack of information : general view on how time and distance at certain attraction point (vertice on the graph) would affect **the next step which the people staying at that point would make.**

-Probability function(flux of people): after plotting the ratio of people who are active respect to intervals of time, (not staying moving from one vertice to another) we draw a continuous curve based on the plotted information.

-Attraction rate: this is the new concept that we are going to define at the beginning of the construction of the model. For a rough guess, the attraction rate could be defined as follows in terms of bikes and docks available at each port.

Then, it is my job to prove that this is actually interchangeable and applicable to concept ‘belongingness.’ (need further research)

At the end of the paper, it will contain one or two case studies to show that the model can actually be applied to the real life setting.

Possible References

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