Miscellany

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Some little topics

- The two envelope paradox.
- The sheep problem.
- Presenting subjective assessments to the public.
- Experiment re Kelly criterion.

Then I'll show some slides from the "Luck" lecture.

The two envelope problem

There are two envelopes containing money one has twice as much money than the other no other information. You choose and open one envelope (effectively a random choice) and see the money. You now have the option to take that money, or to take the money in the other envelope. Which should you do?

This is conceptually easy for a Bayesian. You need a prior distribution on "money in the less-money envelope say density $f(x)$. Given observed amount x the posterior probabilities are

$$\mathbb{P}(\tfrac{x}{2}|x) = \frac{f(\frac{x}{2})/2}{f(\frac{x}{2})/2 \ + \ 2f(2x)}$$

$$\mathbb{P}(2x|x) = \frac{2f(2x)}{f(\frac{x}{2})/2 \ + \ 2f(2x)}$$

So the expectation of the money you get **if you switch** is

$$\tfrac{x}{2}P(\tfrac{x}{2}|x) + (2x)P(2x|x)$$

and you can choose to base your decision on whether this expectation is larger than x.

The wrong analysis is to say: because whatever x you see, it's equally likely to be the larger or smaller amount, so

$$\mathbb{P}(2x|x) = \mathbb{P}(\tfrac{x}{2}|x) = 1/2$$

and (using expectation as criterion) this has expectation $5x/4$ so you should always switch.

This is wrong because you need to make a probability model of how your data arises before doing a math calculation with the observed data.

In fact, to a Bayesian this analysis assume the prior density is $f(x) = 1/x$, but this is not a **probability** density.

A math question given to elementary school children.

> *There are 125 sheep and 5 dogs in a flock. How old is the shepherd?*

According to researchers, three quarters of schoolchildren offer a numerical answer to the shepherd problem. In Kurt Reusser's 1986 study, he describes the typical student response:
$125 + 5 = 130$ . . . this is too big,
and 125 - 5 $= 120$ is still too big. . .
while $125/5 = 25$ . . . that works . . . I think the shepherd is 25 years old.

He concludes common sense has deserted these students in their pursuit of a definitive answer.

As a critique of math education this is fine. But is it fair to say the children are irrational (rather than incorrect)? If every math question you have ever seen does have a definite numerical answer, then (as implicit Bayesians) you might reasonably put a very low prior probability on "unanswerable" and so making a wild guess – even if it only has a $1/100$ chance of being right – is indeed the optimal strategy.

How to present subjective expert probability assessments to the public?

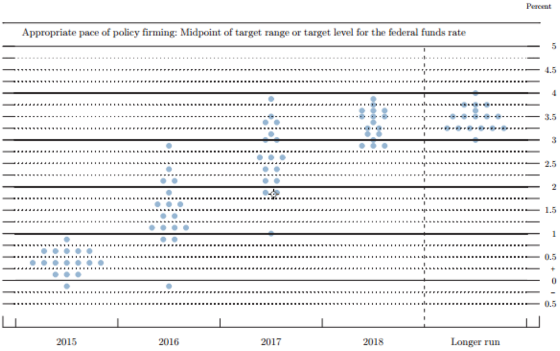
The issue is that each expert has implicitly some probability distribution in mind, but you can't ask them to draw it.

[board – sketch]

How to combine these into one summary? Not clear . . . . . .

What is actually done?

Since 2012, about four times a year the U.S. FOMC (Federal Open Market Committee) members make predictions about where the federal-funds rate will be at the end of the next several years. These predictions are released in the form of "dot-plots" like the one below. Each dot represents one member's prediction for the end of each year. These dots were plotted in September 2015.



Experiment re Kelly criterion