

This relative neighborhood network is part of a family:

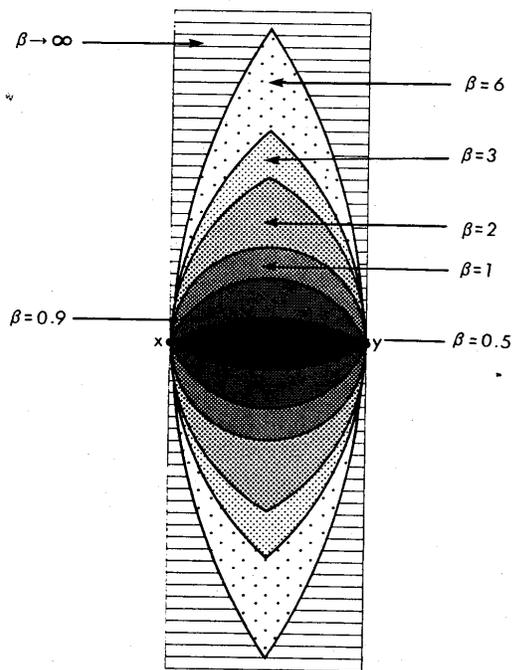
### Proximity graphs

Write  $v_-$  and  $v_+$  for the points  $(-\frac{1}{2}, 0)$  and  $(\frac{1}{2}, 0)$ . The **lune** is the intersection of the open discs of radii 1 centered at  $v_-$  and  $v_+$ . So  $v_-$  and  $v_+$  are not in the lune but are on its boundary. Define a **template**  $A$  to be a subset of  $\mathbb{R}^2$  such that

- (i)  $A$  is a subset of the lune;
- (ii)  $A$  contains the line segment  $(v_-, v_+)$ ;
- (iii)  $A$  is invariant under reflection (left - right and top - bottom)
- (iv)  $A$  is open.

For arbitrary points  $x, y$  in  $\mathbb{R}^2$ , define  $A(x, y)$  to be the image of  $A$  under the transformation (translation, rotation and scaling) that takes  $(v_-, v_+)$  to  $(x, y)$ .

*D.G. Kirkpatrick and J.D. Radke*



**Definition.** Given a template  $A$  and a locally finite set  $\mathbf{x}$  of vertices, the associated **proximity graph**  $G$  has edges defined by: for each  $x, y \in \mathbf{x}$ ,

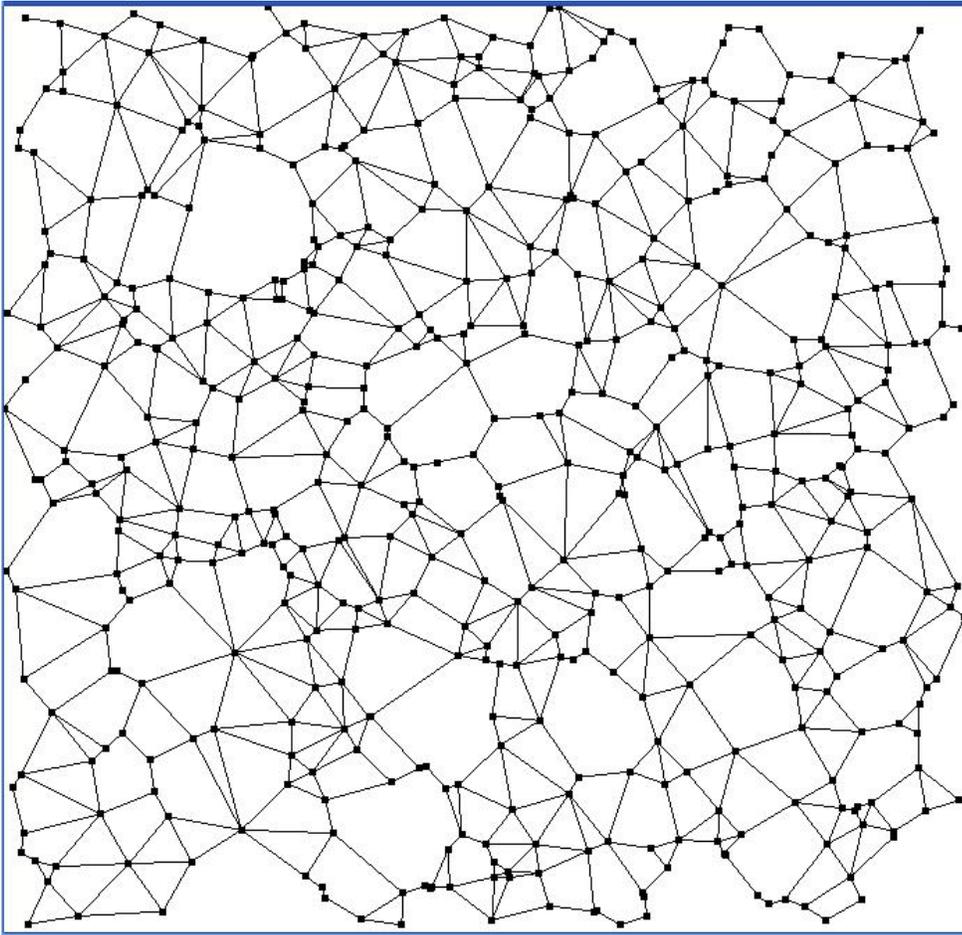
$(x, y)$  is an edge of  $G$  iff  $A(x, y)$  contains no vertex of  $\mathbf{x}$ .

There are two “named” special cases.

If  $A$  is the lune then  $G$  is the **relative neighborhood network**.

If  $A$  is the disc centered at the origin with radius  $1/2$  then  $G$  is called the **Gabriel network**.

Note that replacing  $A$  by a subset  $A'$  can only increase the edge-set.



Gabriel network on 500 cities.