Statistics 153 (Introduction to Time Series) Homework Five

Due on 8 May, 2013

28 April, 2013

1. Consider the following seasonal AR model:

\[(1 - \phi B)(1 - \Phi B^s)X_t = Z_t,\]

where \(\{Z_t\}\) is white noise and \(|\phi| < 1, |\Phi| < 1\).

(a) Calculate the spectral density of \(\{X_t\}\).

(b) Plot the spectral density for \(\phi = 0.5, \Phi = 0.9, \sigma_Z^2 = 1\) and \(s = 12\).

(c) Also plot the spectral density for the AR(1) process \((1 - 0.5B)X_t = Z_t\) and the seasonal AR(1) process \((1 - 0.9B^{12})X_t = Z_t\).

(d) Compare and comment on the different plots.

2. The spectral density of a stationary time series \(\{X_t\}\) is defined on \([-1/2, 1/2]\) by \(f(\lambda) = 5\) for \(1/6 \leq |\lambda| \leq 1/3\) and zero otherwise.

(a) Evaluate the autocovariance function of \(\{X_t\}\) at lags 0 and 1.

(b) Find the spectral density of the process \(\{Y_t\}\) defined by \(Y_t = X_t - X_{t-12}\).

3. Consider the stationary Autoregressive process:

\[X_t - 0.99X_{t-3} = Z_t\]

where \(\{Z_t\}\) is white noise.

(a) Compute and plot the spectral density of \(\{X_t\}\).

(b) Does the spectral density suggest that the sample paths of \(\{X_t\}\) will exhibit approximately oscillatory behaviour? If yes, then with what period?

(c) Simulate a sample of size 100 from this model. Plot the simulated data. Does this plot support the conclusion of part (b)?

(d) Compute the spectral density of the filtered process:

\[Y_t = \frac{X_{t-1} + X_t + X_{t+1}}{3}. \quad (1)\]

How does the spectral density of \(\{Y_t\}\) compare to that of \(\{X_t\}\)?
(e) From the simulated sample from \( \{X_t\} \) in part (c), perform the averaging as in (1) to obtain a simulated sample from \( \{Y_t\} \). Plot this sample. Does this plot support the spectral density plot in part (d)?

4. Without using the `arima.sim()` function in R, simulate \( n = 400 \) observations from the multiplicative seasonal ARMA model given by the difference equation:

\[
(1 - 0.5B)(1 - 0.7B^{12})X_t = Z_t
\]

where \( \{Z_t\} \) is white noise. Plot the sample autocorrelation function of the simulated observations and compare it with the true acf of the process.

5. Consider the first dataset, q1data.R, from the second midterm. Remove the trend and seasonality by differencing first with order 52 and then a usual differencing. Call the resulting dataset \( x_t, t = 1, \ldots, n \) to which a stationary model can be fit.

(a) Estimate the spectral density of \( \{X_t\} \) nonparametrically from the data \( \{x_t\} \).

(b) Fit a reasonable stationary model to \( \{x_t\} \) and estimate the spectral density of \( \{X_t\} \) by the spectral density of the fitted model.

(c) Plot the two estimates of the spectral density on the same plot. Comment on the two plots.