

Homework 6 - Some hints and suggestions

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Problem 3 You may proceed in two ways:

1. Proof by contradiction. Here, you may need to take into account theorem 7.2.13 in page 314 of G&S.
2. Direct proof. Set $\epsilon > 0$ and note that $\exists M > 0 : \mathbb{P}\{X \in [-M, M]\} \leq \epsilon$. Now g will actually be uniformly continuous (equicontinuous) on $[-M, M]$.

Problem 7 Such exercises typically require the use of an inequality and then the application of the first Borel-Cantelli lemma.

Problem 19 For part (a) you could start with the special case that $\mathbb{E}[X_n] = 0$. Then extend your proof to the case $\mathbb{E}[X_n] = \mu_n$. For part (b) recall from theory done earlier, that if every linear combination $aX + bY$ is Normal, then the vector (X, Y) has the bivariate normal distribution.

Problem 20 In order to prove convergence in probability it suffices to prove convergence in \mathcal{L}^2 (convergence in mean square).

Problem 22 Use the strong law of large numbers.