Homework 5 - Some hints and suggestions

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General Remarks

1. It is not necessary that you explicitly calculate the characteristic functions of standard distributions (although it is good practice).

2. Recall that if $\phi_X(t)$ is the characteristic function of X, then $\phi_{aX+b}(t) = e^{ibt}\phi_X(at)$.

3. You can use the following without proof: Suppose that $a_n \to \infty$, while $a_n c_n \to \lambda < \infty$. Then $(1 + c_n)^{a_n} \to e^{\lambda}$.

Problem 21 Express the "conditional" characteristic function as a function of the generating function of Z_n (say G_n). The latter has been given as an example in class for the case of geometric branching (also see example 5.4.3 in Grimmett and Stirzaker).

Problem 23 Let $\phi(t)$ be the cf of X (which is the same as that of Y) and define $\psi(t) := \frac{\phi(t)}{\phi(-t)}$. Follow these steps:

1. Show that $\psi(t) = [\psi(t/2^n)]^{2^n}$.

2. Show that as $t \to 0$ we have $\psi(t) = 1 + o(t^2)$

3. Show $\phi(t) = \phi(-t)$.

Combine the above to get the result.

Problem 39 It might be easier to take the geometric distribution to be defined as the number of *trials* until the first success rather than the number of *failures* until the first success, so that the mass function is $f(x) = pq^{x-1}$, $x \geq 1$ (not that it matters if you take the other definition).

Problem 42 For tidiness, you may assume without loss of generality that $\mu = 0$ and $\sigma^2 = 1$.

Problem 43 For the first part, it's easier to look at the distribution function of Y and differentiate, rather than use the change of variables theorem.