Stat 215B (Spring 2005): Lab 6

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Modelling Seismic Wave Attenuation

When an earthquake occurs, seismic waves radiate out from the source. The amplitude of their oscillation decreases as the hypocentral distance from the source, r, increases. Strong motion seismometers are scattered about and record the maximum horizontal acceleration of the waves, A. A second important quantity derived for each earthquake, is its magnitude, m.

Data on the above quantities are crucial to seismic engineers and architects, who are concerned with the design of buildings likely to withstand earthquakes. They are also important to seismologists interested in the structure of the earth. As an example, it may be required to predict the maximum horizontal acceleration A for a given magnitude m and hypocentral distance r.

Geophysical theory suggests an attenuation law of the form

$$\log A = \alpha + \beta m - \log r + \gamma r,$$

although this is not the only functional form proposed.

Joyner and Boore [*Bulletin of the Seismological Society of America*, 1981] collected data for $I = 23$ California earthquakes. For each particular earthquake i, recordings were collected from a number J_i of seismometers. Thus we have multiple measurements per earthquake.

The data are denoted by A_{ij}, m_i, and d_{ij} ($j = 1, 2, ..., J_i$ and $i = 1, ..., 23$). For example, A_{ij} is the maximum horizontal acceleration for the i-th earthquake as recorded by the j-th seismometer. The variable d is the epicentral distance (in km), and is connected to the hypocentral distance by

$$r = \sqrt{d^2 + 7^2}$$

The data are arranged in four columns: column 1 indexes the event (earthquake), column two provides the magnitudes (m), column three contains the epicentral distance in kilometers (d) and column four contains the maximum horizontal acceleration (A), in units of g (gravitational acceleration).

In this lab you are asked to address the following:

1. Fit and assess the model

$$\log A_{ij} = \alpha + \beta m_i - \log r_{ij} + \gamma r_{ij} + \epsilon_{ij}, \qquad \epsilon_{ij} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2)$$

   What are your assumptions for this model? What does the outcome suggest? Do your assumptions hold?

2. Provide a 99% confidence interval for the maximum horizontal acceleration at an epicentral distance of $d = 50$ km from a magnitude 7.0 earthquake.

3. Consider fitting a robust/resistant version of your model. Is this reasonable based on your previous results? Compare the two models.

4. Now, consider the additive model

$$\log A_{ij} = \phi(m_i) + \gamma(r_{ij}) + \epsilon_{ij}, \qquad \epsilon_{ij} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2),$$

   where $\phi(\cdot)$ and $\gamma(\cdot)$ are unknown but smooth functions. How does this model compare to the models derived from geophysical theory? Quantify this in statistical terms, explaining what you assume and how you proceed.

Note: The data can be found in the file `lab6.txt` on the lab section webpage.