Stat 215B (Spring 2005): The Delta Method

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The one-dimensional case

Suppose that T_n is an estimator of θ, based on a random sample of size n. If

$$\sqrt{n}\left[T_n - \theta\right] \xrightarrow{d} \mathcal{N}\left(0, \sigma^2\right)$$

then

$$\sqrt{n}\left[h\left(T_n\right) - h\left(\theta\right)\right] \xrightarrow{d} \mathcal{N}\left(0, \sigma^2\left[h'\left(\theta\right)\right]^2\right)$$

provided $h'\left(\theta\right)$ exists and is not equal to zero.

The multi-dimensional case

More generally, suppose that $\tilde{\theta} \in \mathbb{R}^m$ and let $\tilde{T_n}$ be an estimator of it, based on a random sample of size n. Let $h : \mathbb{R}^m \to \mathbb{R}^k$ be a mapping defined by $\tilde{\theta} \mapsto (h_1(\tilde{\theta}), ..., h_k(\tilde{\theta}))^T$. If

$$\sqrt{n}\left[\tilde{T_n} - \tilde{\theta}\right] \xrightarrow{d} \mathcal{N}\left(0, \Sigma\right)$$

then

$$\sqrt{n}\left[h\left(\tilde{T_n}\right) - h\left(\tilde{\theta}\right)\right] \xrightarrow{d} \mathcal{N}\left(0, (\nabla h(\tilde{\theta}))^T\ \Sigma\ \nabla h(\tilde{\theta})\right)$$

provided $\nabla h\left(\tilde{\theta}\right)$ exists and is not equal to zero.