Simple, Affordable, Risk-Limiting Election Audits

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[Election Leak] [Voting Machine Wins] [Homer Votes—sort of] Abstract: Abstract: Post-election audits—which compare reported totals with hand counts of a trustworthy audit trail in a random sample of batches—can limit the risk of certifying an incorrect electoral outcome. The risk is the maximum chance the audit stops short of a full hand count when a full hand count would show that the outcome is wrong.

Risk-limiting audits can be built as sequential tests: Data are collected. If they give strong evidence that the outcome is right, the audit stops. Otherwise, more data are collected. Eventually, the audit stops or there has been a full hand count. The strength of the evidence can be quantified using the maximum *P*-value of the hypothesis that the outcome is wrong given the audit data. The maximum is over all ways in which the outcome could be wrong—a nonparametric hypothesis. The *P*-value depends on the sampling scheme, the choice of test statistics, the number of overstatement errors in each audited batch, and more.

Abstract, cont'd

California Secretary of State Debra Bowen is introducing the first legislation in the country to require risk-limiting audits. Though many details remain to be worked out, it appears that the California bill will incorporate sampling with probability proportional to an error bound on the relative margin and will use the Kaplan-Markov test statistic, a fully nonparametric approach that relies on a sampling design and transformation used in financial auditing. The calculation of the *P*-value uses a result of Kaplan that combines a Martingale inequality and Markov's inequality.

There have been six risk-limiting audits, all in California in 2008 and 2009: Marin County (a small measure in February 2008 requiring a supermajority and a county-wide measure in November 2008), Santa Cruz County (County Supervisor, District 1, November 2008), and Yolo County (bond measure in November 2008 and a land-use measure and Community Service District director contest in November 2009). I designed these audits, which were conducted in collaboration with elections officials in the counties. Several sampling techniques were tested. The audits ensured at least a 75All were confirmed at the first stage without full hand counts.

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Abstract, cont'd

Many lessons were learned. Clear, precise and timely communication between the auditors and the elections officials is crucial. While efficiency matters, statistical optimality is less important than simplicity. The biggest barrier to wide-scale risk-limiting audits is the inability of current election management systems (vote tabulation systems) to export data in a useful, machine-readable format. Adopting standard terminology and data formats and common names for contests and candidates across jurisdictions would be extremely helpful. Workload when the electoral outcome is correct scales roughly linearly in batch size. Huge reductions in workload are available—factors of hundreds if vote tabulation systems could report cast vote records mapped to individual physical ballots, making "single-ballot auditing" possible.

Humboldt County CA, 2008

Serious Error in Diebold Voting Software Caused Lost Ballots in California County, by Kim Zetter

Election officials in a small county in California discovered by chance last week that the tabulation software they used to tally votes in this year's general election dropped 197 paper ballots from the totals at one precinct. The system's audit log also appears to have deleted any sign that the ballots had ever been recorded.

Premier has acknowledged ... its software caused the system to delete votes. The company has apparently known about the problem since 2004 ...

[RoV] Crnich would never have discovered the problem through her standard canvassing procedures . . . nor would she have discovered it while conducting a mandatory manual audit that California counties are required to do.

Crnich discovered the missing ballots only because she happened to implement a new and innovative auditing system this year that was spearheaded by members of the public who helped her develop it.

Wired News, 8 December 2008 http://blog.wired.com/27bstroke6/2008/12/unique-election. html

SoS Bowen's response

SACRAMENTO Secretary of State Debra Bowen today announced she has withdrawn state approval of Premier Election Solutions Global Election Management System (GEMS) version 1.18.19, which contains serious software flaws.

Premier GEMS 1.18.19 contains the "Deck Zero" anomaly, a software error that can delete the first batch of optically scanned ballots under certain circumstances without alerting elections officials to the deletion. In addition, the systems audit logs fail to record important events and clear buttons permit deletion of key records, both of which violate federal standards. ...

"Clearly, a voting system that can delete ballots without warning and doesnt leave an accurate audit trail should not be used in California or anywhere," said Secretary Bowen, Californias chief elections officer.

http://www.sos.ca.gov/admin/news-releases.htm

Polk County NC, 2008

Owens victory in Polk is in doubt, by Times-News staff

Ted Owens went to sleep Tuesday night thinking he had earned another term ... A recount Wednesday showed he may not have. ...

Computer software initially displayed figures that were different than those shown by the voting machines ...

The software installed in the stand-alone computer that ballot results are fed into was the problem . . . [Elections Director Dale Edwards] said there was no explanation as to why the computer counted the wrong numbers, and no one is at fault.

BlueridgeNow.com Times-News, 6 November 2008 http://www.blueridgenow. com/article/20081106/NEWS/811050255

Santa Clara County, CA, 2008

Few problems reported in area despite record turnout, by Karen de Sá and Lisa Fernandez

Record-high voting in the Bay Area on Tuesday mostly defied predictions of unwieldy waits and overwhelmed polls. But in Santa Clara County, concerns about touch-screen voting machines will likely increase following significant malfunctions.

Fifty-seven of the county's Sequoia Voting Systems machines failed on Election Day, resulting in hourslong delays before replacements arrived.

Mercury News, 4 November 2008 http://www.mercurynews.com/elections/ ci_10901166?nclick_check=1

Leon County, FL, 2008

Ballots not being recorded at two Leon County polling places, by Angeline J. Taylor

Leon County Supervisor of Elections Ion Sancho has reported that ballots . . . are not being read properly. The problem, he said, rests with a new machine that has been purchased for polling sites throughout the state. . . .

"Certain ballots are being rejected across the state," he said. ... If the machine reads the ballot card as too long, the ... machine will simply not read the card.

Tallahassee Democrat, 20 October 2008 http://www.tallahassee.com/ article/20081020/BREAKINGNEWS/81020024

Palm Beach County, FL, 2008

Florida Primary Recount Surfaces Grave Voting Problems One Month Before Presidential Election, by Kim Zetter

At issue is an August 26 primary election in which officials discovered, during a recount of a close judicial race, that more than 3,400 ballots had mysteriously disappeared after they were initially counted on election day. The recount a week later, minus the missing ballots, flipped the results of the race to a different winner.

... officials found an additional 227 ballots that were never counted on election day ... in boxes in the county's tabulation center.

Palm Beach County was using new optical-scan machines that it recently purchased from Sequoia Voting Systems for \$5.5 million.

Palm Beach County, FL, 2008, cont'd

[In a re-scan of ballots the machines had rejected] [o]fficials expected the machines would reject the same ballots again. But that didn't happen. During a first test of 160 ballots, the machines accepted three of them. In a second test of 102 ballots, the machines accepted 13 of them ... When the same ballots were run through the machines again, 90 of the ballots were accepted.

[T]he county then re-scanned two batches of 51 ballots each that had initially been rejected for having no vote cast in the judicial race, but that were found in a manual examination to contain legitimate votes for one candidate or the other. The first batch of 51 ballots were found to have legitimate votes for Abramson. The second batch of 51 ballots were found to have legitimate votes for Wennet.

In the first batch of 51 ballots ... 11 of the ballots that had previously been rejected as undervotes were now accepted ... the remaining 40 ballots were rejected as having no votes. In the second batch of 51 ballots ... the same machine accepted 2 ballots and rejected 49.

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Palm Beach County, FL, 2008, cont'd

The same two batches of ballots were then run through the second ... machine. [I]n the first batch ... the machine accepted 41 ... and rejected 10 others. In the second batch ... the machine accepted 49 of the ballots and rejected 2—the exact opposite of the results from the first machine.

Wired News, 7 October 2008, http://blog.wired.com/27bstroke6/2008/10/florida-countys.
html

Washington, DC, 2008

Report Blames Speed In Primary Vote Error; Exact Cause of Defect Not Pinpointed, by Nikita Stewart

Speed might have contributed to the Sept. 9 primary debacle involving thousands of phantom votes, according to a D.C. Board of Elections and Ethics report issued yesterday. ... [T]he report does not offer a definitive explanation...

The infamous Precinct 141 cartridge "had inexplicably added randomly generated numbers to the totals that had been reported," according to the report written by the elections board's internal investigative team.

... 4,759 votes were reflected instead of the actual 326 cast there.

Washington Post, 2 October 2008; Page B02

see also hearings at http://www.octt.dc.gov/services/on_demand_ video/channel13/October2008/10_03_08_PUBSVRC_2.asx New Jersey 2008

County finds vote errors: Discrepancies discovered in 5% of machines, by Robert Stern

Five percent of the 600 electronic voting machines used in Mercer County during the Feb. 5 presidential primary recorded inaccurate voter turnout totals, county officials said yesterday ...

23 February 2008, New Jersey Times

Ohio 2004

Machine Error Gives Bush Thousands of Extra Ohio Votes, by John McCarthy

COLUMBUS, Ohio – An error with an electronic voting system gave President Bush 3,893 extra votes in suburban Columbus, elections officials said. Franklin County's unofficial results had Bush receiving 4,258 votes to Democrat John Kerry's 260 votes in a precinct in Gahanna. Records show only 638 voters cast ballots in that precinct. Bush's total should have been recorded as 365.

5 November 2004, Associated Press

Florida 2004

Broward Machines Count Backward, by Eliot Kleinberg

[E]arly Thursday, as Broward County elections officials wrapped up after a long day of canvassing votes, something unusual caught their eye. Tallies should go up as more votes are counted. Thats simple math. But in some races, the numbers had gone ... down.

Officials found the software used in Broward can handle only 32,000 votes per precinct. After that, the system starts counting backward.

... The problem cropped up in the 2002 election. ... Broward elections officials said they had thought the problem was fixed.

5 November 2004, The Palm Beach Post

Problem: Any way of counting votes makes mistakes.

If there are enough mistakes, apparent winner could be wrong.

If there's a complete, accurate audit trail, can ensure big chance of fixing wrong outcomes.

Crucial question: when to *stop* counting, not where to start.

Solution: If there's compelling evidence that outcome is right, stop; else, audit more.

Current audit laws have the wrong focus: Virtually useless for fixing wrong outcomes. (California has something good in the works.)

Efficiency is primarily about batch sizes: Need data plumbing.

California Elections Code §15360

[T]he official conducting the election shall conduct a public manual tally of the ballots tabulated by those devices, including absent voters' ballots, cast in 1 percent of the precincts chosen at random by the elections official ...

The elections official shall use either a random number generator or other method specified in regulations . . .

The official conducting the election shall include a report on the results of the 1 percent manual tally in the certification of the official canvass of the vote. This report shall identify any discrepancies between the machine count and the manual tally and a description of how each of these discrepancies was resolved ...

NJ S507 [1R] (Gill)

[officials] shall conduct random hand counts of the voter-verified paper records in at least two percent of the election districts where elections are held for federal or State office ...

Any procedure designed, adopted, and implemented by the audit team shall be implemented to ensure with at least 99% statistical power that for each federal, gubernatorial or other Statewide election held in the State, a 100% manual recount of the voter-verifiable paper records would not alter the electoral outcome reported by the audit

[procedures] shall be based upon scientifically reasonable assumptions ... including but not limited to: the possibility that within any election district up to 20% of the total votes cast may have been counted for a candidate or ballot position other than the one intended by the voters[.]

Say what?

Others

Oregon and New Mexico have audit laws that allow the sample (of races and/or ballots) to be selected before the election.

Maryland's pending legislation has elaborate tables of sample sizes.

Rep. Rush Holt has proposed federal legislation that has tiered sampling fractions, depending on the margin—but no requirement for followup if errors are found.

Legislation to enunciate *principles*, not *methods*.

Methods best left to regulation, room to improve, fix, etc.

(But political expediency might dictate otherwise—c.f. current CA situation.)

Wrong Focus

Current and proposed laws focus on how big an initial sample to draw. (Again, c.f. CA.)

Heated debates over fixed percentages, tiered percentages depending on the margin, or sample sizes that vary continuously with the margin and depend on batch sizes.

The real issue isn't where to start. It's when to stop.

Can't fix wrong outcomes without counting the whole audit trail.

Risk-Limiting Audits

If the outcome is wrong, there's a at least a [pre-specified] minimum chance of a full manual count, no matter what caused the outcome to be wrong.

The *risk* is the maximum chance that there won't be a full hand count when a full hand count would show that the apparent outcome is wrong.

"Wrong" means disagrees with what a full hand count would show: presupposes accurate & complete audit trail, commitment to semi-official results, secure chain of custody, etc. Nontrivial.

Null hypothesis: outcome is wrong. Control Type I error rate. Role of statistics: Less counting when the outcome is right, but big chance of a full hand count when outcome is wrong.

Persistent idea that only the initial sample matters, not the errors the sample finds. E.g., Holt bill. Essential that voters create complete, durable, accurate audit trail.

Essential that voting systems enable auditors to access reported results (total ballots, counts for each candidate, registered voters) in auditable batches.

Essential to select batches at random, *after* the results are posted. (Can supplement with "targeted" samples.)

Need a plan for dealing with discrepancies, possibly leading to full count. "Explaining" or "resolving" isn't enough.

Current audit laws do not limit risk.

Compliance audits vs. materiality audits.

Assessing Evidence

How strong is the evidence that the outcome is correct, given how the sample was drawn, the margin, the errors found, etc.?

What is the biggest chance that—if the outcome is wrong the audit would have found as little error as it did?

(The definition of "little" differs across sampling methods, etc.)

P-value of the hypothesis that the apparent outcome of one or more contests is wrong.

Notation

N batches (possibly single ballots), C contests.

Contest c has K_c "candidates," votes for up to f_c candidates.

Reported vote for candidate k in batch p is v_{kp}

$$V_k \equiv \sum_{p=1}^N v_{kp}.$$

 \mathcal{W}_c : indices of apparent winners of contest c.

 \mathcal{L}_c : indices of apparent losers of contest c.

Reported margin of $w \in \mathcal{W}_c$ over $\ell \in \mathcal{L}_c$:

$$V_{w\ell} \equiv V_w - V_\ell > 0. \tag{1}$$

More notation

Actual vote for candidate k in batch p is a_{kp} .

 $A_k \equiv \sum_{p=1}^N a_{kp}.$

Actual margin of $w \in \mathcal{W}_c$ over $\ell \in \mathcal{L}_c$:

$$A_{w\ell} \equiv A_w - A_\ell. \tag{2}$$

Apparent winners of all C contests are the true winners iff

$$\min_{c \in \{1,\dots,C\}} \min_{w \in \mathcal{W}_c, \ell \in \mathcal{L}_c} A_{w\ell} > 0.$$
(3)

Still more notation . . .

For $w \in \mathcal{W}_c$, $\ell \in \mathcal{L}_c$, define $e_{pw\ell} \equiv \begin{cases} \frac{(v_{wp} - v_{\ell p}) - (a_{wp} - a_{\ell p})}{V_{w\ell}}, & \text{if ballots in batch } p \text{ contain contest } c \\ 0, & \text{otherwise.} \end{cases}$ (4)

If any apparent outcome is wrong,

$$\exists (c \in \{1, \ldots, C\}, w \in \mathcal{W}_c, \ell \in \mathcal{L}_c) \text{ with }$$

$$\sum_{p=1}^{N} e_{pw\ell} \ge 1.$$
(5)

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Test based on sufficient condition

Define

$$e_p \equiv \max_{c} \max_{w \in \mathcal{W}_c, \ \ell \in \mathcal{L}_c} e_{pw\ell}.$$
 (6)

All outcomes must be correct if

$$E \equiv \sum_{p=1}^{N} e_p < 1.$$
(7)

Maximum across-contest relative overstatement of margins (MACRO)

Controlling the familywise error rate

C null hypotheses,

the outcome of contest c is incorrect, c = 1, ..., C.

If E < 1, the entire family of C null hypotheses is false: all apparent outcomes are right.

Test of hypothesis $E \ge 1$ at significance level α is a test of the *C* hypotheses with familywise error rate no larger than α .

Bounding e_p

If number of valid ballots cast in batch p for contest c is at most b_{cp} then

$$e_{pw\ell} \le (v_{wp} - v_{\ell p} + b_{cp})/V_{w\ell},\tag{8}$$

and so

$$e_p \le \max_{c \in \{1,\dots,C\}} \max_{w \in \mathcal{W}_c, \ell \in \mathcal{L}_c} \frac{v_{wp} - v_{\ell p} + b_{cp}}{V_{w\ell}} \equiv u_p.$$
(9)

 u_p is a limit on the relative overstatement of *any* margin that can be concealed in batch p, the MACRO in batch p.

 $U \equiv \sum_p u_p$, bound on total error.

Sampling Designs

Simple

Stratified (by county, voting method, other)

PPEB

NEGEXP

Stratified PPEB?

Sampling scheme affects choice of test statistic—analytic tractability

Weighted max, binning for simple & stratified sampling, NEG-EXP, PPEB.

More efficient choices possible for PPEB: Kaplan-Markov

Taint

 e_p : error in batch p (max % overstatement of any margin) u_p : upper bound on e_p ; $U = \sum_p u_p$.

The *taint* of batch p is

$$\tau_p = \frac{e_p}{u_p} \le 1. \tag{10}$$

Draw batches with replacement s.t. in each draw

$$\mathbf{P}\{\text{draw batch } p\} = u_p/U. \tag{11}$$

Taint of *j*th draw is T_j . $\{T_j\}$ are iid, $\mathbb{E}T_j = E/U$.

Can stop the audit if can reject the hypothesis $\mathbb{E}T_j \ge 1/U$.

Hypothesis about the mean of a bounded random variable.

Sequential risk-limiting audit using Kaplan-Markov bound

0. Calculate error bounds $\{u_p\}$, U. Set n = 1. Pick $\alpha \in (0, 1)$ and m > 0.

1. Draw a batch using PPEB. Audit it if it has not already bee audited.

2. Find $T_n \equiv t_p \equiv e_p/u_p$, taint of the batch p drawn at stage n.

3. Compute

$$P_n \equiv \prod_{j=1}^n \frac{1 - 1/U}{1 - T_j}.$$
 (12)

4. If $P_n < \alpha$, stop; report apparent outcomes. If n = m, audit remaining batches. If all batches have been audited, stop; report known outcomes. Else, $n \leftarrow n + 1$ and go to 1.

This sequential procedure is risk-limiting:

If any outcome is wrong,

 $\mathbb{P}\{\text{stop without auditing every batch}\} < \alpha$.

Chance $\geq 1 - \alpha$ of fixing wrong outcomes by full hand count.

Remarkably efficient if batches are not too big.

Pilot Audits in California

Marin County (February 2008; November 2008, 2009)

Yolo County (November 2008, 2009)

Santa Cruz County (November 2008)

Measures requiring super-majority, simple measures, multicandidate contests, vote-for-n contests.

Contest sizes ranged from about 200 ballots to 121,000 ballots.

Counting burden ranged from 32 ballots to 7,000 ballots.

Cost per audited ballot ranged from nil to about \$0.55.

2008 Yolo County, CA Measure W Audit





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2009 Yolo County, CA Measure P Audit

| | | Child State | | | (AAA | (A |
|-----|---|-------------|-----|---|------------------------------|-------------|
| | Special Election November 2009 | | | | | |
| | City of Davis | | | Enocial Election November 20 | 09 | |
| | November 03, 2009 | Precin | | City of Davis | | |
| | | | _ | November 03, 2009 | | Precinct 10 |
| | Instruction Text | | | | | |
| | Please use a black or blue ink pen to mark your choices on the ballot | | = | Total | | |
| | To vote for your choice in each contest, completely fill in the box | | | Please use a black or blue ink pen to mark | your choices on the ballot. | |
| | provided to the left of your choice. | | | To vote for your choice in each contest, cor | npletely fill in the box | |
| | MEASURE P | | | provided to the left of your choice. | | |
| | Shall Resolution No. 09-132, amending the Davis General Plan to | | = | MEASURE P | Davis General Plan to | |
| | acticulture to residential uses, as set forth in the Resolution and | | | change the land use designations for the W | ildhorse Ranch property from | |
| | establishing the Base Line Project Features for development of the | | | agriculture to residential uses, as set forth in | the Resolution and | |
| 0 | Wildhorse Ranch Project be approved? | | _ | establishing the Base Line Project Features Wildhorse Ranch Project be approved? | | |
| Ž | Yes | | 6 | | | |
| ŏ I | | | 30 | L Yes | | |
| 21 | L NO | | 00 | No No | | |
| 21 | | | 01 | | | |
| Z | | | 17 | | | |
| 5 | | | 8 | | | |
| 31 | | | 8 | | | |
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Yolo County Measure P, November 2009

| Reg. voters | ballots | precincts | batches | yes | no |
|-------------|---------|-----------|---------|-------|-------|
| 38,247 | 12,675 | 31 | 62 | 3,201 | 9,465 |

(VBM) and in-person (IP) ballots were tabulated separately (62 batches).

U = 3.0235.

For $\alpha = 10\%$, initial sample size 6 batches; gave 4 distinct batches, 1,437 ballots.

Single-ballot auditing would save *lots* of work

Can determine the initial sample size for a Kaplan-Markov single-ballot audit even though the cast vote records (CVRs) were not available.

For $\alpha = 10\%$ would need to look at CVRs for n = 6 ballots.

For $\alpha = 1\%$, n = 12 ballots.

C.f., 1,437 ballots for actual batch sizes.

Director, Esparto Community Service District, Yolo County

Voters could select up to f = 2 candidates.

1 precinct; 988 registered voters; 187 ballots cast.

| Reg. | ballots | Jordan | Pomeroy | Fescenmeyer | Moreland | under | over |
|--------|---------|--------|---------|-------------|----------|-------|-------|
| voters | | | | | | votes | votes |
| 988 | 187 | 95 | 80 | 64 | 62 | 57 | 8 |

Esparto, contd.

The smallest margin 80 - 64 = 16 votes.

Did not have CVRs so could not compute sharp u_p s. Pessimistic assumption $u_p = 0.125$ for every ballot. $U = 187 \times 0.125 = 23.375$.

Initial sample n = 32 ballots, for $\alpha = 25\%$.

If mean u_p for sample were true for all 187, U = 16.874. Then:

n = 23 would have sufficed to limit the risk to $\alpha = 25\%$.

n = 32 would give $\alpha = 14.2\%$.

California legislation is coming

Conference call yesterday with Assistant Chief Deputy Secretary of State Jennie Bretschneider and folks from Verified Voting.

California Secretary of State is drafting legislation requiring risk-limiting audits.

Expected to call for risk-limiting audits of *all* contests.

Serious logistical issues: MACRO? Batch sizes? Coordination across jurisdictions? Uniform labeling of contests and candidates across jurisdictions?

Data plumbing is crucial.

Law likely to be based on PPEB and the Kaplan-Markov bound.

What do we need for efficient audits?

Laws that allow/require risk-limiting audits, but mostly ...

Data plumbing:

Structured, small batch data export from VTSs.

A way to associate individual CVRs with physical ballots.

Reducing counting effort is mostly about reducing batch sizes.

Extra slides (time is unlikely to permit)

Cartoon

| | | | | | | | IP bat | ches | VBM ba | atches |
|---|-----------|---------|---------|--------|--------|--------|--------|-------|--------|--------|
| | precincts | batches | ballots | winner | loser | margin | winner | loser | winner | loser |
| Α | 200 | 400 | 120,000 | 60,000 | 54,000 | 6,000 | 200 | 180 | 100 | 90 |
| В | 100 | 200 | 60,000 | 30,000 | 24,000 | 6,000 | 200 | 160 | 100 | 80 |
| С | 60 | 120 | 36,000 | 18,000 | 12,600 | 5,400 | 200 | 140 | 100 | 70 |

Contest A: entire jurisdiction, 200 precincts.

Contest B: 100 precincts.

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Contest C: 60 precincts; 30 of those are also in contest B.
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Each precinct is divided into two batches, 400 ballots cast

in-precinct (IP) and 200 ballots cast by mail (VBM).

Valid votes, undervotes, and invalid ballots.

| Cartoon. | contd. |
|-------------|---------|
| Car 00 011, | 0011001 |

| | | | | FWER | | | | PCER | |
|-----|-------|----|----------|-----------|-----------|----|----------|-----------|-----------|
| | | | expected | expected | expected | | expected | expected | expected |
| | U | n | batches | ballots | votes | n | batches | ballots | votes |
| А | 21.00 | 52 | 48.49 | 16,074.23 | 16,074.23 | 33 | 31.58 | 10,488.77 | 10,488.77 |
| В | 11.00 | 28 | 26.01 | 8,615.69 | 8,615.69 | 17 | 16.27 | 5,402.16 | 5,402.16 |
| С | 7.67 | 19 | 17.50 | 5,795.81 | 5,795.81 | 12 | 11.41 | 3,787.51 | 3,787.51 |
| all | | | 85.13 | 28,038.26 | 30,485.73 | | 56.38 | 18,649.98 | 19,678.44 |
| Μ | 22.72 | 36 | 34.30 | 11,387.29 | 20,617.68 | | | | |

Independent and simultaneous audits controlling FWER and PCER to risk $\alpha = 25\%$.

The bottom row is MACRO

Cartoon, contd.

| α | Single- | Batch audit | |
|----------|---------|--------------|-----------|
| | sharp | conservative | |
| 25% | 39.99 | 60.98 | 9,878.64 |
| 10% | 66.97 | 101.96 | 16,065.45 |
| 1% | 132.90 | 202.83 | 29,566.79 |

Expected initial sample sizes, in ballots.

MACRO

Maximum across-contest relative overstatement in batch p:

$$e_p \equiv \max_{c \in \{1, \dots, C\}} \max_{w \in \mathcal{W}_c, \ell \in \mathcal{L}_c} e_{pw\ell}.$$
 (13)

Now

$$\max_{c \in \{1,...,C\}} \max_{w \in \mathcal{W}_c, \ell \in \mathcal{L}_c} \sum_{p=1}^{N} e_{pw\ell} \leq \sum_{p=1}^{N} \max_{c \in \{1,...,C\}} \max_{w \in \mathcal{W}_c, \ell \in \mathcal{L}_c} e_{pw\ell}$$

$$= \sum_{p=1}^{N} e_p \equiv E.$$
(15)

E is maximum across-contest relative overstatement (MACRO).

If E < 1, all C apparent outcomes are right.