

The Will of the People and the Luck of the Draw: Using Statistics to Limit the Risk of Wrong Electoral Outcomes

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Many collaborators, including:

Josh Benaloh, Joe Hall, Mike Higgins, Doug Jones, Eric Lazarus, Mark Lindeman, Luke Miratrix, Olivier Pereira, Ron Rivest, David Wagner, Dan Wallach, Kai Wang, Vince Yates.

Lots of help from elections officials, especially: Jennie Bretschneider, Elaine Ginnold, Neal Kelley, Freddie Oakley, Tom Stanionis.

Lots of help and support from election integrity advocates, especially: Susannah Goodman, David Jefferson, Pam Smith

Special shout-out to Steve Pierson, ASA



The right to vote is the right from which all other rights spring. -???

It doesn't matter who votes. What matters is who counts the votes. – Josef Stalin

The purpose of elections is to convince the losers that they lost. – Dan Wallach

The difference between theory and practice is smaller in theory than it is in practice. – Various

Software maker takes blame in Wellington vote count mess, by George Bennett

The supplier of Palm Beach County's voting and tabulating equipment says a software "shortcoming" led to votes being assigned to the wrong candidates and the elections office declaring the wrong winners in two recent Wellington council races. ... Unbeknownst to elections officials, the vote totals for the mayor's race ended up being reported and later certified as the results of the Seat 1 race. The Seat 1 vote totals were certified as the Seat 4 results and the Seat 4 vote totals were certified as the mayoral results.

The problem wasn't discovered until six days after the election, during a routine audit. ... The fact that the audit is conducted after winners are certified is a requirement of state law.

THE PALM BEACH POST, 23 MARCH 2012, http:

News

//www.sun-sentinel.com/news/palm-beach/pb-bucher-election-machines-20120323,0,7453964.story

Saguache County, Colorado crimes

Grand jury has its hands full with Saguache election case, by Troy Hooper

A disputed election in south-central Colorado is now in the hands of a grand jury that is reviewing allegations that the clerk and other officials committed crimes when they tallied the votes.

The officials under investigation stood to benefit from the election's outcome — most notably Saguache County Clerk Melinda Myers — who, along with County Commissioner Linda Joseph, at first lost but then won their races after Myers declared the races had to be retabulated due to a technical glitch.

[Myers won't let the Colorado Secretary of State inspect the ballots.] "There are processes that we are avowed to protect," [Colorado County Clerks] association president Scott Doyle said. "One of them is preserving the sanctity of ballots. The cornerstone of our democracy is based on those ballots. It's what we stand for as clerks."

"The clerks are using the false argument about 'secrecy of ballots' as a scare tactic or sympathy evoking tool to try to get a trusting public to side with them in their effort to block public verification of elections," Al Kolwicz of the Colorado Voter Group said in an email. "Why exactly clerks oppose public verification is unknown."

Officials in Saguache County stand accused of more than 30 misdemeanors. [Myers was recalled this year by a 60% vote.]

THE COLORADO INDEPENDENT, 25 MARCH 2011,

http://coloradoindependent.com/80819/grand-jury-has-its-hands-full-with-saguache-election-case

Waukesha County, WI: Oops!

Wisconsin Election Surprise: David Prosser Gains 7,500 Votes After 'Human Error' In Waukesha County, by Amanda Terkel

In a dramatic turn of events on Thursday, the Waukesha County clerk announced that the vote total announced for Tuesday's Wisconsin Supreme Court race had been mistaken – and that the corrected numbers changed the outcome of the entire election.

There were 3,456 missing votes for Democratic-backed challenger JoAnne Kloppenburg and 11,059 for incumbent GOP-backed Justice David Proser. Kloppenburg has previously been beating Prosser by just 200 votes of the roughly 1.5 million cast statewide.

In the city of New Berlin, the total for one ward was recorded as 37 votes for Prosser, but it was actually 237, she said. In the town of Lisbon, a "typing error" resulted in both candidates losing votes. The most significant error, however, occurred in the city of Brookfield.

"The spreadsheet from Brookfield was imported into a database that was provided by the Government Accountability Board, but it inadvertently was not saved," Nickolaus said. "As a result, when I ran the report to show the aggregate numbers that were collected from all the municipalities, I assumed that the city of Brookfield was included. It was not. The city of Brookfield cast 14,315 votes on April 5 – 10.859 votes went for Justice David Prosser, 3,456 went for JoAnne Kloppenburg,"

... prior to the election, Nickolaus "was heavily criticized for her decision to keep the county results on an antiquated personal computer, rather than upgrade to a new data system being utilized statewide."

"Nickolaus cited security concerns for keeping the data herself"

HUFFINGTON POST, 7 APRIL 2011,

http://www.huffingtonpost.com/2011/04/07/david-prosser-wisconsin-supreme-court_n_846431.html

Vote-flipping in North Carolina

NC GOP leader: Touchscreen voting machines have programming flaw, by Michael Biesecker

The chairman of the N.C. Republican Party alleged Thursday that a programming flaw with touchscreen voting machines used for early voting in 36 counties is causing votes intended for GOP candidates to be counted for Democrats.

Tom Fetzer, the Republican chairman, said that if the State Board of Elections does not enact a list of demands intended to remedy the problem by the end of today, the party's lawyers will be in federal court Friday morning seeking a statewide injunction. ...

Johnnie McLean, deputy director of the state elections board, said Thursday that her office has received no widespread reports of problems.

"In every election we will have scattered reports of machines where the screens need to be recalibrated," McLean said. "That sort of comes with the territory with touch-screen technology."

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NEWS OBSERVER, 28 OCTOBER 2010, http://www.newsobserver.com/2010/10/28/766257/ nc-republican-party-chair-touchscreen.html#ixzz13gTJCCvp

Humboldt County CA, 2008

Serious Error in Diebold Voting Software Caused Lost Ballots in California County, by Kim Zetter

Election officials in a small county in California discovered by chance last week that the tabulation software they used to tally votes in this year's general election dropped 197 paper ballots from the totals at one precinct. The system's audit log also appears to have deleted any sign that the ballots had ever been recorded.

Premier has acknowledged ... its software caused the system to delete votes. The company has apparently known about the problem since 2004 ...

[RoV] Crnich would never have discovered the problem through her standard canvassing procedures ... nor would she have discovered it while conducting a mandatory manual audit that California counties are required to do.

Crnich discovered the missing ballots only because she happened to implement a new and innovative auditing system this year that was spearheaded by members of the public who helped her develop it.

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WIRED NEWS, 8 DECEMBER 2008, http://blog.wired.com/27bstroke6/2008/12/unique-election.html

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Polk County NC, 2008

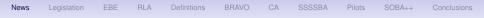
Owens victory in Polk is in doubt, by Times-News staff

Ted Owens went to sleep Tuesday night thinking he had earned another term ... A recount Wednesday showed he may not have. ...

Computer software initially displayed figures that were different than those shown by the voting machines ...

The software installed in the stand-alone computer that ballot results are fed into was the problem ... [Elections Director Dale Edwards] said there was no explanation as to why the computer counted the wrong numbers, and no one is at fault.

BLUERIDGENOW.COM TIMES-NEWS, 6 NOVEMBER 2008, http: //www.blueridgenow.com/article/20081106/NEWS/811050255



Santa Clara County, CA, 2008

Few problems reported in area despite record turnout, by Karen de Sá and Lisa Fernandez

Record-high voting in the Bay Area on Tuesday mostly defied predictions of unwieldy waits and overwhelmed polls. But in Santa Clara County, concerns about touch-screen voting machines will likely increase following significant malfunctions.

Fifty-seven of the county's Sequoia Voting Systems machines failed on Election Day, resulting in hourslong delays before replacements arrived.

MERCURY NEWS, 4 NOVEMBER 2008, http:

//www.mercurynews.com/elections/ci_10901166?nclick_check=1

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Ballots not being recorded at two Leon County polling places, by Angeline J. Taylor

Leon County Supervisor of Elections Ion Sancho has reported that ballots . . . are not being read properly. The problem, he said, rests with a new machine that has been purchased for polling sites throughout the state. . . .

"Certain ballots are being rejected across the state," he said. ... If the machine reads the ballot card as too long, the ... machine will simply not read the card.

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TALLAHASSEE DEMOCRAT, 20 OCTOBER 2008, http://www.tallahassee.com/article/20081020/BREAKINGNEWS/81020024



Florida Primary Recount Surfaces Grave Voting Problems One Month Before Presidential Election, by Kim Zetter

At issue is an August 26 primary election in which officials discovered, during a recount of a close judicial race, that more than 3,400 ballots had mysteriously disappeared after they were initially counted on election day. The recount a week later, minus the missing ballots, flipped the results of the race to a different winner.

... officials found an additional 227 ballots that were never counted on election day ... in boxes in the county's tabulation center.

Palm Beach County was using new optical-scan machines that it recently purchased from Sequoia Voting Systems for \$5.5 million.

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Palm Beach County, FL, 2008, cont'd

[In a re-scan of ballots the machines had rejected] [o]fficials expected the machines would reject the same ballots again. But that didn't happen. During a first test of 160 ballots, the machines accepted three of them. In a second test of 102 ballots, the machines accepted 13 of them ... When the same ballots were run through the machines again, 90 of the ballots were accepted.

[T]he county then re-scanned two batches of 51 ballots each that had initially been rejected for having no vote cast in the judicial race, but that were found in a manual examination to contain legitimate votes for one candidate or the other. The first batch of 51 ballots were found to have legitimate votes for Abramson. The second batch of 51 ballots were found to have legitimate votes for Wennet.

In the first batch of 51 ballots ... 11 of the ballots that had previously been rejected as undervotes were now accepted ... the remaining 40 ballots were rejected as having no votes. In the second batch of 51 ballots ... the same machine accepted 2 ballots and rejected 49.



Palm Beach County, FL, 2008, cont'd

The same two batches of ballots were then run through the second ... machine. [I]n the first batch ... the machine accepted 41 ... and rejected 10 others. In the second batch ... the machine accepted 49 of the ballots and rejected 2—the exact opposite of the results from the first machine. WIRED NEWS, 7 OCTOBER 2008, http:

//blog.wired.com/27bstroke6/2008/10/florida-countys.html

Washington, DC, 2008

Report Blames Speed In Primary Vote Error; Exact Cause of Defect Not Pinpointed, by Nikita Stewart

Speed might have contributed to the Sept. 9 primary debacle involving thousands of phantom votes, according to a D.C. Board of Elections and Ethics report issued yesterday. ... [T]he report does not offer a definitive explanation...

The infamous Precinct 141 cartridge "had inexplicably added randomly generated numbers to the totals that had been reported," according to the report written by the elections board's internal investigative team.

... 4,759 votes were reflected instead of the actual 326 cast there.

WASHINGTON POST, 2 OCTOBER 2008; PAGE B02

see also hearings at

http://www.octt.dc.gov/services/on_demand_video/ channel13/October2008/10_03_08_PUBSVRC_2.asx



County finds vote errors: Discrepancies discovered in 5% of machines, by Robert Stern

Five percent of the 600 electronic voting machines used in Mercer County during the Feb. 5 presidential primary recorded inaccurate voter turnout totals, county officials said yesterday ...

23 FEBRUARY 2008, NEW JERSEY TIMES



Machine Error Gives Bush Thousands of Extra Ohio Votes, by John McCarthy

An error with an electronic voting system gave President Bush 3,893 extra votes in suburban Columbus, elections officials said. Franklin County's unofficial results had Bush receiving 4,258 votes to Democrat John Kerry's 260 votes in a precinct in Gahanna. Records show only 638 voters cast ballots in that precinct. Bush's total should have been recorded as 365.

5 NOVEMBER 2004, ASSOCIATED PRESS

News Legislation EBE RLA Definitions BRAVO CA SSSSBA Pilots SOBA++ Conclusions Florida 2004

Broward Machines Count Backward, by Eliot Kleinberg

[E]arly Thursday, as Broward County elections officials wrapped up after a long day of canvassing votes, something unusual caught their eye. Tallies should go up as more votes are counted. Thats simple math. But in some races, the numbers had gone ... down.

Officials found the software used in Broward can handle only 32,000 votes per precinct. After that, the system starts counting backward.

... The problem cropped up in the 2002 election. ... Broward elections officials said they had thought the problem was fixed.

5 NOVEMBER 2004, THE PALM BEACH POST



California Elections Code §15360

[T]he official conducting the election shall conduct a public manual tally of the ballots tabulated by those devices, including absent voters' ballots, cast in 1 percent of the precincts chosen at random by the elections official ...

The elections official shall use either a random number generator or other method specified in regulations ...

The official conducting the election shall include a report on the results of the 1 percent manual tally in the certification of the official canvass of the vote. This report shall identify any discrepancies between the machine count and the manual tally and a description of how each of these discrepancies was resolved ...

NJ S507 [1R] (Gill)

Legislation

[Officials] shall conduct random hand counts of the voter-verified paper records in at least two percent of the election districts where elections are held for federal or State office ...

Any procedure designed, adopted, and implemented by the audit team shall be implemented to ensure with at least 99% statistical power that for each federal, gubernatorial or other Statewide election held in the State, a 100% manual recount of the voter-verifiable paper records would not alter the electoral outcome reported by the audit ...

[Procedures] shall be based upon scientifically reasonable assumptions . . . including but not limited to: the possibility that within any election district up to 20% of the total votes cast may have been counted for a candidate or ballot position other than the one intended by the voters[.]

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Oregon and New Mexico have audit laws that allow the sample (of races and/or ballots) to be selected before the election.

Florida does not allow auditing before results are final; limits the amount of auditing.

Rep. Rush Holt has proposed federal legislation that has tiered sampling fractions, depending on the margin—but no requirement for followup if errors are found.

Can't correct wrong outcomes without counting the whole audit trail.

Counting the whole audit trail won't give right answer unless it's adequately intact.

What should an election audit law do?

Legislation should enunciate *principles*, not *methods*.

Methods are best left to regulation: Easier to improve, fix, etc.

Mutual distrust among election integrity advocates, elections officials, and legislators is an unfortunate but important consideration.



Strongly Software-Independent Voting System (Rivest & Wack)

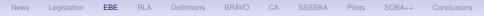
A voting system is strongly software-independent if an undetected error or change to its software cannot produce an undetectable change in the outcome, and we can find the correct outcome without rerunning the election.

Risk-limiting Audit

Large, known chance of a full hand count if the outcome is wrong, thereby correcting the outcome.

Risk is maximum chance of failing to correct an apparent outcome that is wrong, no matter what caused the outcome to be wrong.

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Evidence-based elections

Evidence = Auditability + Auditing.

Resilient Canvass Framework

Known minimum chance that the overall system (human, hardware, software, procedures) gives the correct election outcome—when it gives an outcome.

- Use voting system that creates a voter-verified audit trail.
- Conduct a compliance audit to ensure that—as actually used in this election—the system is strongly software-independent.
- If so, conduct a risk-limiting audit. If not, do not declare an outcome.

Overall election and canvass process should correct its own errors before reporting, or report it can't guarantee that it corrected its errors (e.g., because audit trail can't be shown to be intact).

Role of certification of voting systems

- 1. Under laboratory conditions, can the vote tabulation system—as delivered from the manufacturer—count votes with a specified level of accuracy?
- 2. As maintained, deployed, and used in the current election, did the vote tabulation system find the true winners?

In U.S., certification can cost millions and take years. Addresses Q 1.

Q 2 seems more important. Audits address Q 2.

If a jurisdiction uses a certified system, costs more to use it as a component of a resilient canvass framework because auditing will be more expensive.

Moreover, audit is less transparent.

Maintenance costs high; systems not agile; stupefying inertia

Certification still useful for some things, e.g., to ensure accessibility and creation of durable audit trail.

Compliance Audits and Materiality Audits

Effective compliance audit

Determine whether the audit trail is trustworthy enough to determine who won.

If not, do not declare an outcome (nb: danger of DOS attacks).

Effective materiality audit

Correct (with high probability) the outcome if it is wrong.

Requires intact audit trail–need to pass compliance audit first. Might require counting the entire audit trail by hand.

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- Historically, much debate over how large a sample to start with. Detection paradigm.
- If we want audits to correct wrong outcomes, crucial question is when to *stop* auditing.
- Answer: If there's compelling evidence that outcome is right, stop; else, audit more.
 Measure evidence by *P*-value.
- Eventually, either have strong evidence that the outcome is right, or the whole contest has been counted by hand and correct outcome is known.
- Sequential test of the null hypothesis that the outcome is wrong. "Risk" is chance of type I error: concluding a wrong outcome is right. Can control rigorously. No possibility of a type II error.



Role of statistics

Limiting the risk is easy

No statistics needed: just count all the ballots by hand.

Statistics lets you do less counting when the outcome is right, but still ensure a big chance of a full hand count when outcome is wrong.

Ballot-polling audits and Comparison Audits

- Ballot polling audit: sample ballots until there is strong evidence that looking at all of them would show the same election outcome.
- Comparison audit:
 - 1. Commit to vote data at some level of aggregation.
 - 2. Check that the committed data produces the same results as claimed. Should be perfect.
 - 3. Sample the committed data and check until there is strong evidence that it is accurate enough to find the right election outcome.

Tradeoffs

- Ballot polling audit
 - Virtually no set-up costs
 - Requires nothing of voting system
 - Generally, need a ballot manifest to draw sample
 - Preserves voter anonymity except possibly for sampled ballots
 - Counting burden comparable to precinct-based comparison audit unless margin is small
 - · Requires more counting than ballot-level comparison audit
 - Does not check tabulation: could be right because of lucky cancellation of errors
- Comparison audit
 - Heavy demands on voting system for reporting and export
 - Requires LEO to commit to subtotals
 - Requires ability to retrieve ballots that correspond to CVRs or subtotals
 - May compromise voter privacy (small-batch or ballot-level reporting) & enable coercion through pattern voting
 - Most efficient (ballot-level) may require re-scanning all ballots
 - Checks tabulation (but not for *transitive audits* unless subtotals are cross checked as well)

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Ballot-level comparison audits require least hand counting

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- 16 pilot audits in CA, CO, and OH; another 14 planned.
- EAC funding for pilots in CA and CO and Cuyahoga County, OH
- CO has law; CA has pilot law
- simple measures
- measures requiring super-majority
- multi-candidate contests
- vote-for-n contests,
- multiple contests audited simultaneously with one sample

- contest sizes: 200 ballots to 121,000 ballots
- counting burden: 16 ballots to 7,000 ballots
- cost per audited ballot: nil to about \$0.55.

California AB 2023 (Saldaña, sponsored by SoS Bowen)

Unanimous bipartisan support in both houses.

11 counties committed to pilots; 20 interested.

(b)(3) "Risk-limiting audit" means a manual tally employing a statistical method that ensures a large, predetermined minimum chance of requiring a full manual tally whenever a full manual tally would show an electoral outcome that differs from the outcome reported by the vote tabulating device for the audited contest. A risk-limiting audit shall begin with a hand tally of the votes in one or more audit units and shall continue to hand tally votes in additional audit units until there is strong statistical evidence that the electoral outcome is correct. In the event that counting additional audit units does not provide strong statistical evidence that the electoral outcome is correct, the audit shall continue until there has been a full manual tally to determine the correct electoral outcome of the audited contest.

http://www.leginfo.ca.gov/pub/09-10/bill/asm/ab_ 2001-2050/ab_2023_bill_20100325_amended_asm_v98.html

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- *Risk-limiting audit*: pre-specified minimum chance of correcting apparent outcome if apparent outcome is wrong. (Endorsed by ASA, CC, VV, LWV, CEIMN, ...)
- *Risk*: largest possible chance an apparent outcome that's wrong won't be caught and corrected—no matter why it's wrong.
- *Simultaneous risk-limiting audit*: pre-specified minimum chance of correcting all incorrect apparent outcomes in the election.
- *Simultaneous risk*: largest possible chance that one or more wrong outcomes won't be caught and corrected—no matter why they are wrong.

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- How strong is the evidence that the outcome is correct, given how the sample was drawn, the margin, etc.?
- What is the biggest chance that—if the outcome is wrong—the audit would have found what it did?

• (Maximum) *P*-value of the hypothesis that the apparent outcome of one or more contests is wrong.

Wald ballot-polling audit, 2 candidates, risk limit α

- Pick *D*, maximum draws before full hand count. *s* is winner's share of the valid votes according to the vote tabulation system.
 Set *T* = 1, *d* = 0.
- 2. Select a ballot at random from ballots cast in the contest.
- If the ballot is an undervote, overvote, or an invalid ballot, return to step 2; else d ← d + 1.
- 4. If the ballot shows a valid vote for the reported winner, multiply T by

s/50%.

5. If the ballot shows a valid vote for anyone else, multiply T by

$$(1 - s)/50\%$$
.

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- If T > 1/α, stop the audit: Reported outcome stands. Else if d < D, return to step 2.
- 7. Perform full hand count; hand-count results trump reported results.

Theorem: limits risk to α .

Actual ballot-polling audit in Monterey County, CA

Conducted in Monterey County in May, 2011, before certification. 10% risk limit

Expected number of ballots to examine: 58.

Actual: 92 draws (89 distinct ballots).

Workload

Winner's	Quantiles					
True Share	25 th	50 th	75 th	90 th	99 th	Mean
70%	12	22	38	60	131	30
65%	23	38	66	108	236	53
60%	49	84	149	244	538	119
58%	77	131	231	381	840	184
55%	193	332	587	974	2,157	469
54%	301	518	916	1,520	3,366	730
53%	531	914	1,619	2,700	5,980	1,294
52%	1,188	2,051	3637	6,053	13,455	2,900
51%	4,725	8,157	14,486	24,149	53,640	11,556
50.5%	18,839	32,547	57,838	96,411	214,491	46,126

Means and percentiles of number of ballots with valid votes to inspect for 10% risk limit using BRAVO, as a function of the winner's share of vote, 2-candidate contest (estimated using 10⁷ replications)

General case: C-candidate, k-winner contest

Test that every winner $w \in W$ beat every loser $\ell \in \mathcal{L}$. k(C-k) null hypotheses: loser ℓ beat winner w.

Test w/ same sample but different test statistics $\{T_{w\ell}\}$.

Define $s_{w\ell} \equiv s_w/(s_w + s_\ell)$, fraction of votes *w* was reported to have received among ballots reported to show a vote for *w* or ℓ or both.

Can be calculated from standard reported election results.

Define $\pi_{w\ell}$ to be actual fraction of votes *w* received among ballots that show a vote for *exactly one* of $\{w, \ell\}$.

Assertion and Sufficient Condition

 $\forall w \in \mathcal{W}, \ell \in \mathcal{L}:$

- If *w* reportedly beat ℓ , $s_{w\ell} > 0.5$.
- If w actually beat ℓ , $\pi_{w\ell} > 0.5$.

BRAVO for C-candidate k-winner contest

- 1. Pick *D*. Set d = 0 and set $T_{w\ell} = 1$ for all $w \in W$ and $\ell \in \mathcal{L}$.
- 2. Draw a ballot uniformly at random with replacement from those cast in the contest. $d \leftarrow d + 1$.
- 3. If the ballot shows a valid vote for a reported winner *w*, then for each ℓ in \mathcal{L} that did not receive a valid vote on that ballot multiply $T_{w\ell}$ by $s_{w\ell}/0.5$. Repeat for all such *w*.
- 4. If the ballot shows a valid vote for a reported loser ℓ , then for each w in \mathcal{W} that did not receive a valid vote on that ballot multiply $T_{w\ell}$ by $(1 s_{w\ell})/0.5$. Repeat for all such ℓ .
- For all (w, ℓ) with T_{wℓ} ≥ 1/α, reject null hypothesis that ℓ beat w. Do not update those T_{wℓ} further.
- 6. If all null hypotheses have been rejected, stop: The reported results stand.

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Else if d < D, return to step 2.

7. Perform full hand count; results replace the reported results.

Theorem: Limits risk to at most α .

Multiplicity in pairwise testing for *k*-winner contest

Stopping short of a full hand count is an error only if at least one of the null hypotheses is in fact true.

Stops short of full hand count only if all k(C - k) null hypotheses are rejected.

Consider the set of null hypotheses that are true. Chance of erroneously rejecting *all* of those is at most the smallest chance of erroneously rejecting any individually.

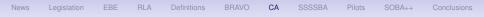
Hence, by testing every (winner, loser) pair individually at level α , the chance of stopping short of a full hand count if any of the C - k apparent losers actually won is at most α .

Moreover, works simultaneously for any number of contests, using the same sample.



For 255 state presidential contests between 1992 and 2008, the median expected sample size to confirm the plurality winner in each state using BRAVO was 307 ballots (per state).

2008 Presidential election in California could have been verified at 10% risk by examining about 100 ballots statewide (in expectation).



Comparison audits: MACRO

Error: Hand-count disagrees with reported count; hand-count presumed correct.

Overstatement: correcting the error would narrow at least one margin. Increase the required sample—decrease confidence.

Understatement: correcting the error would widen every margin. Decrease required sample—increase confidence—but by less.

More confidence if sample shows no misstatements than if understatements balance overstatements.

Sufficient condition for all outcomes to be right:

For every (winner, loser) pair, net overstatement of the margin between them is less than 100% of the reported margin between them.



Bounding the maximum of sums by the sum of maxima leads to nonparametric testing problem:

Is the mean of a nonnegative population less than 1?

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Sampling Designs

- Most jurisdictions that have audits use stratified cluster sampling.
- For most certified systems, limited to some kind of cluster sample (c.f., audits in Alameda, Humboldt, Merced, Monterey, Napa, Orange, San Luis Obispo, Stanislaus, Yolo).
- Simple, Stratified (by county, voting method, other), PPEB/PPS, NEGEXP, Stratified PPEB?
- Sampling scheme affects choice of test statistic—analytic tractability
- Weighted max, binning for simple & stratified sampling, NEGEXP, PPEB.
- More efficient choices possible for PPEB: Kaplan-Markov, Feige?

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Sequential risk-limiting audit using Kaplan-Markov bound

- 1. Calculate error bounds $\{u_p\}$, *U*. Set d = 1. Pick $\alpha \in (0, 1)$ and D > 0.
- 1. Draw a batch using PPEB. Audit batch if it has not already been audited.
- 3. Find $T_d \equiv t_p \equiv e_p/u_p$, taint of the batch *p* drawn at stage *d*.
- 4. Compute

$$P_d \equiv \prod_{j=1}^d \frac{1 - 1/U}{1 - T_j}$$
. See November 2010 WIRED, p.56 (1)

 If P_d < α, report apparent outcomes and stop. If d = D, audit remaining batches, report then-known outcomes and stop.. Else, d ← d + 1 and go to 2.

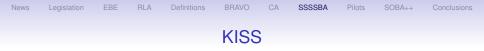
This sequential procedure is risk-limiting

Chance \geq 1 $-\,\alpha$ of correcting wrong outcomes by full hand count

If any outcome is wrong,

 $\mathbb{P}\{\text{stop short of full hand count}\} < \alpha.$

Remarkably efficient if batches are not too big.



Exploit statistical efficiency of *ballot-level auditing*, which compares CVR with human interpretation of individual ballots.

- Audit entire collection of contests with one simple random sample of ballots.
- Very simple calculation determines when to stop.
- Chance of correcting all wrong outcomes is guaranteed to be at least 1 α .
- Transparent, easy to observe.
- Only have to count to 1 (for plurality contests): does ballot have vote for a candidate, or not?
 (A ballot can agree with CVR or have overstatement or understatement of 1 or 2 votes.)

Super-simple++ audit

10% risk limit. *m* is "diluted margin."

 o_1 , o_2 , u_1 , u_2 are 1 and 2-vote overstatements and understatements in the sample

- Pick *D*, maximum draws before full hand count. *s* is winner's share of the valid votes according to the vote tabulation system. Set *T* = 1, *d* = 0.
- 2. Select a ballot at random, uniformly, from ballots cast in the contest. $d \leftarrow d + 1$.
- Compare ballot to CVR; note whether correct, understatement, overstatement
- 4. If $d \ge \frac{4.8+1.4(o_1+5o_2-0.6u_1-4.4u_2)}{m}$, stop audit: reported results stand Else if d < D, return to step 2.
- 5. Perform full hand count; hand-count results trump reported results.

Theorem: limits risk to α .

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Secret sauce

 To implement ballot-level comparison audits, have to associate individual cast vote records (CVRs) with individual physical ballots.

Impossible with current U.S. federally certified systems.

 "Transitive" auditing using an unofficial vote tabulation system that does produce CVRs—such as those of OpenCount or TEVS—and confirming transitively that the apparent outcome is correct, might be the best interim option. (See Calendrino et al. 2007)

If official system says "Lincoln won" and unofficial system says "Lincoln won," then if unofficial system is right, so is official system.

 Performed transitive audits in Alameda, Merced, Napa, Stanislaus, Ventura.
 Orange, Yolo upcoming.

Pilots

Pilot Audits

California: Alameda (4 contests), Humboldt (3 contests), Marin (2 elections, 2 contests total), Merced (2 contests), Monterey (1 contest), Napa (1 contest), Orange (1 contest), San Luis Obispo (2 contests), Santa Cruz (1 contest), Stanislaus (1 contest), Ventura (1 contest), Yolo (2 elections, 3 contests total).

Madera, Orange, Yolo in next few weeks.

Colorado: Boulder County.

Ohio: Cuyahoga County.

Measures requiring super-majority, simple measures, multi-candidate contests. vote-for-*n* contests.

Contest sizes ranged from about 200 ballots to 121,000 ballots.

Counting burden ranged from 17 ballots (to confirm 4 contests simultaneously!) to 7,000 ballots.

Cost per audited ballot ranged from nil to about \$0.55.

2012 Napa County, CA, Supervisor–2nd District



statistics.berkeley.edu/~stark/Vote/auditTools.htm Tools for Comparison Risk-Limiting Election Audits

To hide or show everything but the tools, click this link.

Initial sample size

		(10100).0112.	uted margin: 0.86%	
Contest 1. Contest name: Supervisor,	2nd District			
Vinners: 2 ÷				
Reported votes:				
Candidate 1 Name: Juliana Inman	Vot	tes: 1772]	
Candidate 2 Name: Mark Luce	Vot	tes: 2696		
Candidate 3 Name: Mark Van Gorder	Vot	tes: 1833		
Add candidate to contest 1 Remove last can Add contest Remove last contest	ndidate from contest 1			
Audit parameters				
Risk limit: 10%				
Expected rates of differences (as dec	imal numbers):			
Overstatements. 1-vote: 0.001	2-vote: 0.0001			
Jnderstatements. 1-vote: 0.001	2-vote: 0.0001			

2012 Napa County, CA, Supervisor–2nd District



statistics.berkeley.edu/~stark/Vote/auditTools.htm Random sampling

Pseudo-Random Sample of Ballots	
Seed: 73567556725160627585	
Number of ballots: 7116	
Current sample number: 623	
Draw this many ballots: 623 draw sam	nple reset
Ballots selected: 🗹 show sequence number	s 📄 show hash values
sequence_number, ballot	
1,2086 2,2462	
3,3320	
4,4719	
5,4813	
6,3838	
7,2655	
8,2747 9,3059	
Ballots selected, sorted:	
19,34,37,38,51,90,96,96,99,101,109,114,150,156,163,	
372,395,403,404,407,417,429,444,450,451,471,477,48	
5,596,597,613,614,615,629,645,647,657,685,692,692,	
842,857,862,871,874,876,884,901,906,923,923,934,93 081,1097,1105,1125,1126,1130,1165,1205,1210,1218,	
57.1370.1372.1388.1406.1422.1425.1432.1433.1434.1	
3,1524,1540,1572,1574,1575,1576,1611,1614,1626,16	
1761,1764,1774,1788,1791,1793,1816,1827,1851,1855	5,1893,1921,1978,1989,2010,2017,2034,2056,2
056,2058,2062,2069,2083,2086,2100,2112,2152,2189,	
91,2295,2302,2331,2332,2390,2391,2395,2398,2401,2	422,2436,2462,2463,2474,2495,2513,2514,252
Ballots selected, sorted, duplicates removed	1:
19,34,37,38,51,90,96,99,101,109,114,150,156,163,175	
,403,404,407,417,429,444,450,451,471,477,480,481,44	
13,614,615,629,645,647,657,685,692,694,739,750,763	
,874,876,884,901,906,923,934,937,958,963,973,978,10 ,1130,1165,1205,1210,1218,1219,1224,1226,1284,128	
422.1425.1432.1433.1434.1446.1447.1457.1484.1494.	
75,1576,1611,1614,1626,1634,1638,1642,1644,1665,1	
1,1793,1816,1827,1851,1855,1893,1921,1978,1989,20	
2100,2112,2152,2189,2192,2206,2208,2210,2213,2224	
391,2395,2398,2401,2422,2436,2462,2463,2474,2495,	2513,2514,2520,2549,2556,2558,2563,2578,25

statistics.berkeley.edu/~stark/Vote/auditTools.htm Find ballots using a ballot manifest

-Ballot look-up tool

Ballot manifest: Each line must have a batch label, a comma, and one of the following:

(i) the number of ballots in the batch

(ii) a range specified with a colon (e.g., 131:302), or

(iii) a list of ballot identifiers within parentheses, separated by spaces (e.g., (996 998 1000)).

Each line should have exactly one comma.

001_211161_01,23	
002_211162_02,9	
003_211561_03,32	
004_211561_03,50	
005_211561_03,50	
006_211562_04,14	
007_211562_04,50	
008_211562_04,50	
009_211562_04,50	
010_211563_05,12	
011_211751_06,27	
012_211761_07,2	
013_211761_07,50	
014_211761_07,50	
015_211761_07,50	
016_211761_07,50	
017_211771_08,2	
018_221161_09,16	
019_221161_09,50	
020_221161_09,50	
021_221161_09,50	
022_221162_10,30	
023_221162_10,50	
024_221162_10,50	
025_221162_11,50	

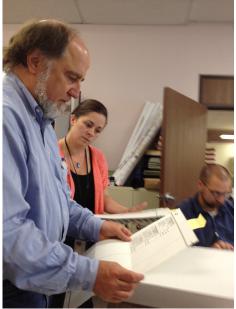
Ballots to look up (separated by commas):

and a Bible Bible

3

2012 Napa County, CA, Supervisor–2nd District

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statistics.berkeley.edu/~stark/Vote/auditTools.htm

look up ballots

Sorted lookup table:

Should more ballots be audited?

Stopping sample size and escalation
Ballots audited so far: 623

I-vote overstatements: 0 Rate: 0
2-vote overstatements: 0 Rate: 0
2-vote understatements: 0 Rate: 0
2-vote understatements: 0 Rate: 0
Estimated stopping size
Catculate Audit complete

News Legislation EBE RLA Definitions BRAVO CA SSSSBA Pilots SOBA++ Conclusions

2008 Yolo County, CA Measure W



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SOBA++

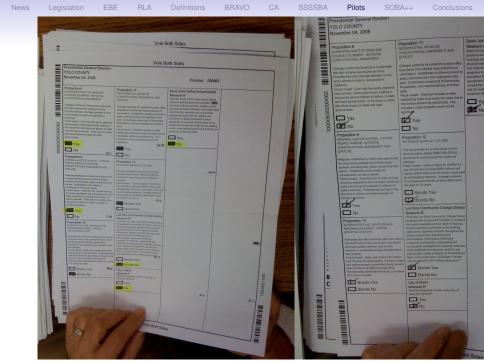
Conclusions

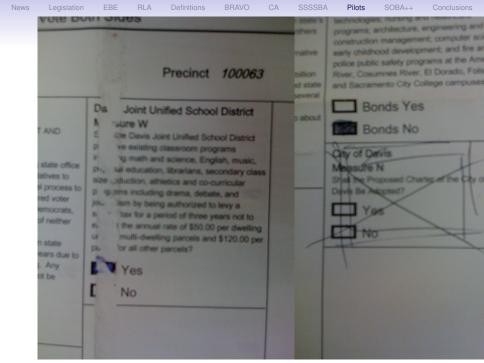
2008 Yolo County, CA Measure W











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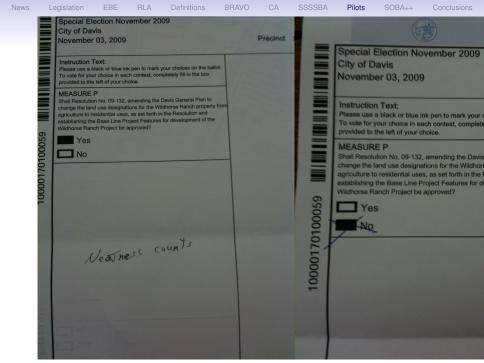
SSSSBA

Pilots

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2009 Yolo County, CA Measure P

Special Election November 2009 City of Davis			Carlos and
November 03, 2009	Precir	Special Election November 2009 City of Davis November 03, 2009	F
Instruction Test: Prese was a black or blue hit pan to mark your chokes on the ballet To reak for your choke in each contest, completely fill in the low provide to the left of your choke. MEASURE P Shall Rescalation No: 58-752, annexing the Duris General Plan to Charge the lense the signature as the Withorse Ranch property from which may be a sub-signation as the the Withorse Ranch property from which may be also the signature as the the Withorse Ranch property from which may be also be approved? Withorse Ranch Project be approved? Version		Instruction Text: Please use a black or blue ink pen to mark your choices on the ballot. To vola for your choice in each contest, completely fill in the box provided to be lift of your choice. MEASURE P Shall Resolution No. 09-132, amending the Davis General Plan to change the land use designations for the Wildhorne Reach property from apticulture to realisetial uses, as all forth. The Reach property from apticulture to realisetial uses, as all forth. The Reach property from apticulture to realisetial uses, as all forth. The Reaching and Wildhorne Rench Project be approved?	
		I Ho	

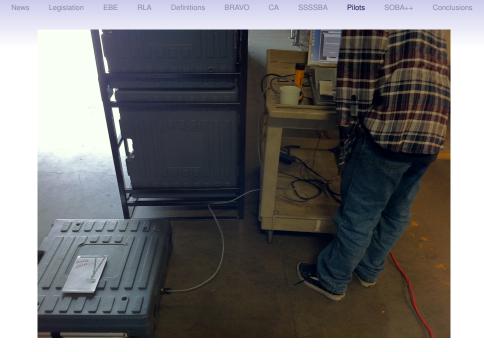


Pilots

2011 Orange County, first audit under AB 2023









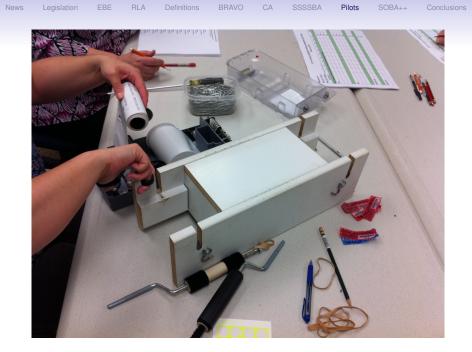
BRAVO CA

CA SSS

Pilots SOE

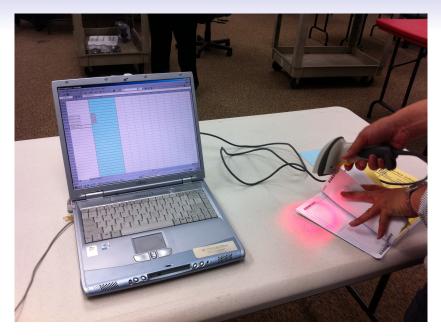
Conclusions

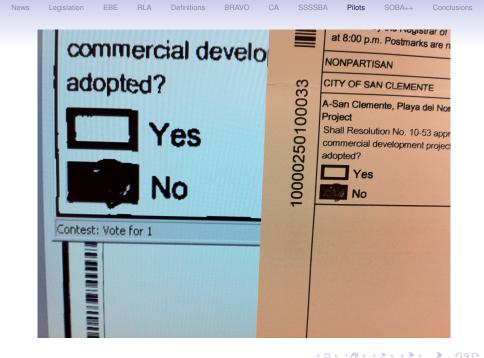




BRAVO CA

Pilots





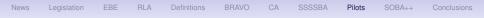
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Conclusions





Risk-limiting Audits: Costs

San Clemente Measure A, 3/8/2011

1% Statutory Audit: \$257.68

Scales as the size of the contest: a contest twice as large would cost about twice as much to audit.

Chance no error found even if outcome was wrong could be 88%.

Risk-limiting: \$483.79 (does not include my time or airfare) Would have cost essentially the same for any contest with the same percentage margin, no matter how large the contest. 90% chance of full hand count if outcome wrong.

SOBA: Preserve voter anonymity, better verifiability

Way to audit that:

- Has a big chance of correcting all wrong outcomes (risk-limiting).
- Enables the public to have strong evidence that the outcome is right, without having to trust (many) others.
- Transparent but preserves voter anonymity—ballot-level data published, but pattern signaling prevented.

• Is efficient, affordable, and currently feasible.

News Legislation EBE RLA Definitions BRAVO CA SSSSBA Pilots SOBA++ Conclusions More Definitions

- *Canvass framework*: the vote-tabulation system together with other human, hardware, software, and procedural components of the canvass, including compliance audit and other audits.
- Canvass framework is *resilient with probability P* or *P-resilient* if the probability that the outcome it gives is the correct outcome is at least *P*, even if its software has an error, shortcoming, or undetected change: System tends to recover from (some) faults. (Strong software independence [Rivest & Wack], plus procedures that exploit that independence.)
- *P*-resilience can mean requiring a re-vote if the audit trail can't be shown to be in good shape.



• Canvass framework is *personally verifiable P-resilient* if it is *P*-resilient and a single individual could, as a practical matter, observe enough of the process to have convincing evidence that the canvass framework is in fact *P*-resilient.

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 Personally verifiable privacy-preserving P-resilient canvass framework: personally verifiable P-resilient and it does not sacrifice privacy unnecessarily.



- Adds a special risk-limiting audit to a strongly software-independent voting system that has had a compliance audit.
- Publishes results by ballot by contest: anybody can verify outcomes.
- Does not allow public to reconstruct whole-ballot CVRs, to protect privacy.
- Uses cryptographic commitment to allow auditors and observers to reconstruct the ballots selected for audit.

• Audit checks accuracy of CVRs *and* of the cryptographic commitment.

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Aside: cryptographic commitments

- Ensures that the ballot identifier is secret but indelible, so every ballot is properly reflected in the electronic results.
- Select and publish commitment function *H*().
- To commit that a given CCVR comes from ballot b, LEO selects secret "salt" u and computes y = H(b, u). Publishes shrouded ID (SID) y.
- If ballot *b* is selected for audit, LEO can reveal *u* and *b*: Anyone can check whether y = H(b, u).



Better ballot accounting

Ballot manifests are not a solved problem.

It's easy to deal with errors in ballot manifest if there's an upper bound on the number of ballots in each container.

But sometimes there isn't a good upper bound—esp. with multipage ballots.

Research directions

- IRV/RCV, NPV
- "False winner rate"
- Extend KM to stratified cluster samples
- Sharper test given sampling design
- Optimal tests if sampling design is up for grabs. Concentration inequalities? Feige?
- Clarify cost/risk tradeoff
- Implement SOBA
- Audit E2E encrypted systems (Wallach, Pereira, et al.)
- Build easily auditable voting systems
- Revise certification requirements?
- Write good legislation for evidence-based elections
- Simpler, simpler, simpler



Resources, further reading

http://www.sos.ca.gov/voting-systems/oversight/
risk-limiting-pilot.htm (AB 2023 pilot program)

http://statistics.berkeley.edu/~stark/Vote (papers, talks, etc.)

http://statistics.berkeley.edu/~stark/Preprints/ RLAwhitepaper-v10.pdf (white paper on RLA legislation)

http://statistics.berkeley.edu/~stark/Vote/auditTools.htm
(simple tool for risk-limiting audits)