Earthquake Clustering and Declustering

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Quake Physics versus Quake Statistics

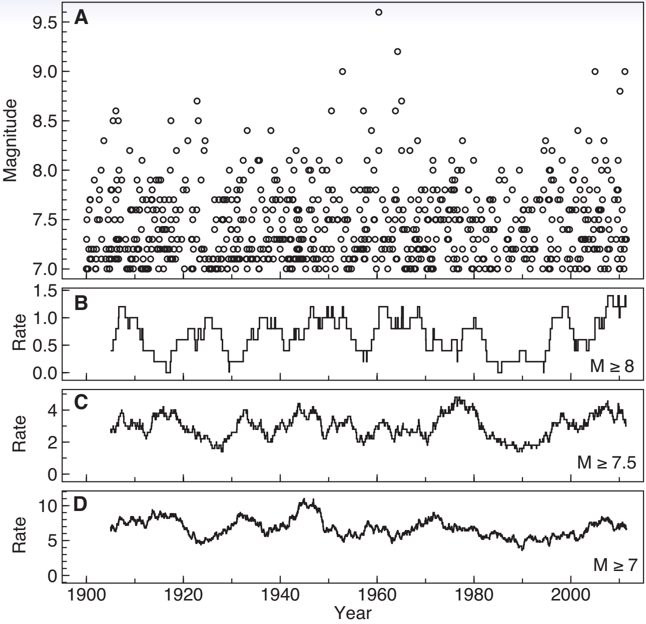
- Distribution in space, clustering in time, distribution of sizes (Gutenberg-Richter law: $N \propto 10^{a-bM}$)
- Foreshocks, aftershocks, swarms—no physics-based definitions
- Clustering makes *some* prediction easy: If there's a big quake, predict that there will be another, close and soon. Not very useful. Cf., today's NY Times `http://www.nytimes.com/2011/10/04/science/04quake.html?_r=1&nl=todaysheadlines&emc=tha210`
- Physics hard: Quakes are gnat's whiskers on Earth's tectonic energy budget
- Spatiotemporal Poisson model doesn't fit at regional scales
- More complex models "motivated by physics"

Has the global risk of large events recently increased?

- 2011 M 9.0 Tohoku-Oki, Japan
- 2010 M 8.8 Maule, Chile,
- 2004 M 9.0 Sumatra-Andaman
- does this reflects change in the underlying process?
- if regional-scale clusters (aftershocks) are removed, are remaining large events noticeably different from a homogeneous Poisson process?

Data

- Moment magnitudes (*Mw*) and times, $M \geq 7$ events
- PAGER-CAT catalog 1900–6/30/2008 (40,767 days)
- PDE and PDE-W catalogs, 7/1/2008–8/13/2011
- remove events preceded by larger events w/i 3 years & 1000 km.


A
B
C
D
Magnitude
Rate
Rate
Rate
M ≥ 8
M ≥ 7.5
M ≥ 7
Year
9.5
9.0
8.5
8.0
7.5
7.0
1.5
1.0
0.5
0.0
4
2
0
10
5
0
1900
1920
1940
1960
1980
2000

Anomalies

- Many $M \geq 8.5$ events, 1950–1965
- Few in 1966–2003
- Elevated rate of $M \geq 8$ earthquakes 2004–, but not of smaller?
- Bufe & Perkins (2011), Perkins (2011), Brodsky (2009): global swarms
- Michael (2011) less impressed

Monte Carlo Tests

- If seismicity is spatially heterogeneous temporally homogeneous Poisson process, conditional marginal distribution of times, given the number of events is iid uniform.
- Estimates based on 100,000 random catalogs with iid uniform times on [0, 40,767], number of events equal to observed.
- Sampling error in estimated P-values on the order of 0.16%.
- Look at specific anomalies and at standard statistical tests of the Poisson hypothesis.

Chance of specific anomalies for iid times

- 9 of 75 $M \geq 8$ events in 2,269 days between 12/23/2004 M 8.1 Macquarie and 3/11/2011 M 9.0 Tohoku-Oki.
- $\approx$ 85% chance that at least 9 of 75 events occur within 2,269 days of each other
- 3 of 16 $M \geq 8.5$ events earthquakes in 2,266 days between 12/26/2004 M 9.0 Sumatra and Tohoku-Oki.
- $\approx$ 97% chance that at least 3 of 16 events occur within 2,266 days of each other.
- 3 of 6 $M \geq 8.8$ events occur in 2,266-days.
- $\approx$ 14% chance.
- No $M \geq 8.5$ events in the $\sim$40 years between 2/4/1965 and 12/26/2004 is more anomalous than the recent elevated rate.
- $\approx$ 1.3% chance of such a long gap—but feature chosen in retrospect. There's always something anomalous.

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Poisson dispersion test

- Divide time [0, 40,767] into $N_w = 100$ intervals.
- Times are conditionally IID, so events are independent “trials” with 100 possible outcomes.
- Chance event falls in each interval is equal
- Joint distribution of counts in intervals multinomial.
- Expected number in each interval is $n/100$.
- Chi-square statistic proportional to sample variance of counts.
- Calibrate by simulation rather than chi-square approximation

Multinomial chi-square test

- Divide time [0, 40,767] into $N_w = 100$ intervals.
- In each interval, count of events unconditionally Poisson.
- Estimate rate λ of Poisson from observed total but pretend rate known a priori

$$K^{-} \equiv \min\left\{k : \; N_w e^{-\lambda} \sum_{j=0}^{k} \lambda^j / j! \geq 5\right\}.$$

$$K^{+} \equiv \max\left\{k : \; N_w \left(1 - e^{-\lambda} \sum_{j=0}^{k-1} \lambda^j / j!\right) \geq 5\right\}.$$

- 1 and 7 for the 330 $M \geq 7.5$ events
  0 and 2 for 75 $M \geq 8.0$ events.

Multinomial chi-square, continued

Define

$$E_k \equiv \begin{cases} N_w e^{-\lambda} \sum_{j=0}^{K^-} \lambda^j / j!, & k = K^- \\ N_w e^{-\lambda} \lambda^k / k!, & k = K^- + 1, \ldots, K^+ - 1 \\ N_w (1 - e^{-\lambda} \sum_{j=0}^{K^+ - 1} \lambda^j / j!), & k = K^+. \end{cases}$$

$$X_k \equiv \begin{cases} \text{\# intervals with} \leq K^- \text{ events}, & k = K^- \\ \text{\# intervals with } k \text{ events}, & k = K^- + 1, \ldots, K^+ - 1 \\ \text{\# intervals with} \geq K^+ \text{ events}, & k = K^+. \end{cases}$$

Test statistic

$$\chi^2 \equiv \sum_{k=K^-}^{K^+} (X_k - E_k)^2 / E_k.$$

Calibrate by simulation rather than chi-square approximation.

Multinomial chi-square test limitations

- Relies on approximation that can be poor.
- Ignores ignores spatial distribution.
- Ignores order of the K intervals: invariant under permutations.
- For instance, the chi-square statistic would have the same value for counts $(N_k) = (3, 1, 0, 2, 0, 4, 1, 0)$ as for counts $(N_k) = (0, 0, 0, 1, 1, 2, 3, 4)$. The latter hardly looks Poisson.
- Hence, chi-square has low power against some alternatives.

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Kolmogorov-Smirnov Test

- Test whether, conditional on the number of events, re-scaled times are iid $U[0,1]$.

$$\text{KS statistic } (U[0,1] \text{ null}): \; D_n = \sup_t \left| \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}(t_i \leq t) - t \right| .$$

- Doesn't require estimating parameters or ad hoc Nw, K^-, K^+, $\hat{\lambda}$.

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Power against alternatives

- KS: long-term rate variations
- Poisson dispersion test (conditional chi-square): heterogeneity across intervals
- Multinomial chi-square: departure from Poisson distribution across intervals
- Poisson dispersion and Multinomial chi-square insensitive to the order of the intervals: rearrangements don't matter
- KS and Poisson dispersion would not reject for equispaced events; Multinomial would, with enough data: under-dispersed.

magnitude threshold	removed	events	p-value KS	PD	MC
7.5	none	444	22.9%	24.1%	62.0%
	AS	330	94.0%	88.8%	10.0%
	AS, FS	268	82.3%	95.1%	56.3%
8.0	none	82	33.8%	79.1%	25.7%
	AS	75	60.3%	89.4%	22.3%
	AS, FS	72	49.0%	89.8%	34.4%

Estimated p-values from 100,000 random catalogs. SE$\approx$ 0.16%.

No statistical evidence for clustering and no physical theory that would lead to clustering on global scales.

Conclusion: risk not elevated.

Why decluster?

Online FAQ for USGS Earthquake Probability Mapping Application:

Q: "Ok, so why do you decluster the catalog?"

A: "to get the best possible estimate for the rate of mainshocks"

"the methodology requires a catalog of independent events (Poisson model), and declustering helps to achieve independence."

- What's a mainshock?
- Aren't foreshocks and aftershocks potentially destructive?

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“Main events,” “foreshocks,” and “aftershocks”

- An event that the declustering method does not remove is a main shock.
- An event that the declustering method removes is a foreshock or an aftershock.

…profound shrug …

Where’s the physics?

Declustering Methods

- Window-based methods
  - Main-shock window: punch hole in catalog near each “main shock”
  - Linked window: every event has a window.
    Clusters are maximal sets of events such that each is in the window of some other event in the group.
    Replace cluster by single event: first, largest, “equivalent”

  Generally, larger events have larger space-time windows
- Stochastic methods: use chance to decide which events to keep
- Other methods (e.g., waveform similarity)
- Straw man: deTest.

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Are "main events" Poisson in time?

Gardner & Knopoff, 1974:
"Is the sequence of earthquakes in Southern California, with aftershocks removed, Poissonian?"

Abstract: "Yes."

Statistical test: multinomial chi-square

Easy to make declustered catalogs indistinguishable from Poisson by deleting enough shocks—or by using a weak test. Shrug.

Multinomial chi-square test on a number of declustered catalogs, including a catalog of 1,751 $M \geq 3.8$ events in Southern California, 1932–1971.

Close to SCEC catalog for 1932–1971, not exact (1,556 $M \geq 3.8$ events)

Declustered: 503 events. 10-day intervals. $d = 2$ degrees of freedom. Don't give B; don't explain how λ estimated.

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Tests on simulated data

Process	KS power	mult. chi-square test power
Heterogeneous Poisson	1	0.1658
Gamma renewal	0.0009	1

Estimated power of level-0.05 tests of homogeneous Poisson null hypothesis from 10,000 simulations. Multinomial chi-square test uses 10-day intervals, 4 categories, and $d = 2$ degrees of freedom. “Heterogeneous Poisson”: rate 0.25 per ten days for 20 years, then at rate 0.5 per ten days for 20 years. “Gamma renewal”: inter-event times iid gamma with shape 2 and rate 1.

Methods tested on SCEC data

- GKI: Remove every event in the window of some other event.
- GKIb: Divide the catalog into clusters: include an event in a cluster if and only if it occurred within the window of at least one other event in the cluster. In every cluster, remove all events except the largest.
- Method GKm: Consider the events in chronological order. If the *i*th event falls within the window of a preceding larger shock that has not already been deleted, delete it. If a larger shock falls within the window of the *i*th event, delete the *i*th event. Otherwise, retain the *i*th event.
- RI: Reasenberg's (1985) method
- dT: deTest—remove events deliberately to make the result pass the multinomial chi-square and KS tests. ad hoc; not optimal.

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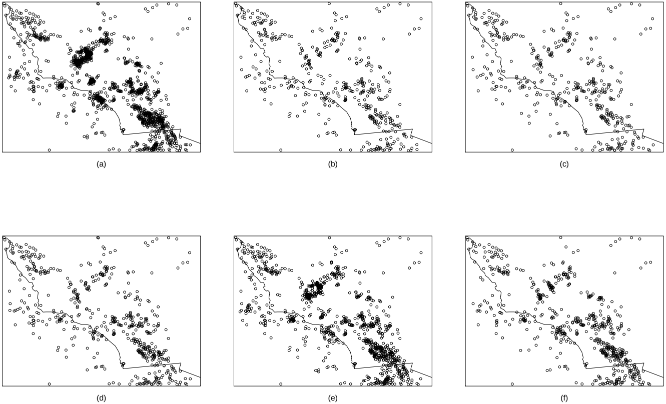
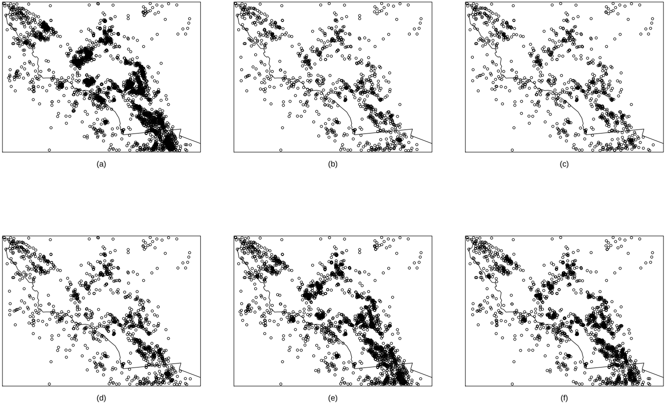
SCEC $M \geq 3.8$, 1932–1971



(a) 1,556 events; (b): The 437 GKl; (c): 424 GKlb. (d): 544 GKm. (e): 985 Rl. (f): 608 dT.

SCEC $M \geq 3.8$, 1932–2010



(a): 3,368 events; (b): 913 GKl; (c): 892 GKlb; (d): 1,120 GKm; (e): 2,046 Rl; (f): 1,615 dT.

Exchangeability of times

- For SITHP, marginal distribution of times is Poisson, so when temporal test rejects, implicitly rejects SITHP.
- For SITHPs, two events can be arbitrarily close. Window declustering imposes minimum spacing, so can't be SITHP.
- For SITHPs, conditional on the number of events, the events are iid with probability density proportional to the space-time rate. Conditional on the locations, the marginal distribution of times is iid, hence exchangeable.

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- For SITHP, marginal distribution of times is Poisson, so when temporal test rejects, implicitly rejects SITHP.
- For SITHPs, two events can be arbitrarily close. Window declustering imposes minimum spacing, so can't be SITHP.
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Exchangeability, contd.

Location of the *i*th event is (x_i, y_i), $i = 1, \ldots, n$.
x_i is longitude, y_i is latitude.

T_i: Time of the event at (x_i, y_i).

Π: Set of all $n!$ permutations of $\{1, \ldots, n\}$.

Process has *exchangeable times* if, conditional on the locations,

$$\{T_1, \ldots, T_n\} \stackrel{d}{=} \{T_{\pi(1)}, \ldots, T_{\pi(n)}\}$$

for all permutations $\pi \in \Pi$.

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Exchangeability, contd.

- SITHP has exchangeable times.
- If events close in space tend to be close in time—the kind of clustering real seismicity exhibits—times not exchangeable.
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Permutation test set up

- $\hat{P}_n$: empirical distribution of the times and locations of the n observed events.
- $\tau(\hat{P}_n)$: projection of $\hat{P}_n$ onto the set of distributions with exchangeable times
  τ puts equal mass at every element of the orbit of data under the permutation group on times.
- $V \subset R^3$ is a *lower-left quadrant* if:

$$V\{x = (x, y, t) \in R^3 : x \leq x_0 \text{ and } y \leq y_0 \text{ and } t \leq t_0\}.$$

- **V**: the set of all lower-left quadrants.

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Test statistic

$$\sup_{V \in \mathbf{V}} |\hat{P}_n(V) - \tau(\hat{P}_n)(V)|$$

- Generalization of the KS statistic to three dimensions.
- Suffices to search a finite subset of **V**.
  Can sample at random from that finite subset for efficiency.
- Calibrate by simulating from $\tau(\hat{P}_n)$—permuting the times (Romano)

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Years	Mag (events)	Meth	n	Multinomial χ^2: χ^2	Multinomial χ^2: Sim	CC	BZ	KS	Romano P	Reject? Time	Reject? Space-time
32–71	3.8 (1,556)	GKl	437	0.087	0.089	0.069	0.096	0.011	0.005	Yes	Yes
		GKlb	424	0.636	0.656	0.064	0.108	0.006	0.000	Yes	Yes
		GKm	544	0	0	0	0	0.021	0.069	Yes	No
		Rl	985	0	0	0	0	0.003	0	Yes	Yes
		dT	608	0.351	0.353	0.482	0.618	0.054	0.001	No	Yes
	4.0 (1,047)	GKl	296	0.809	0.824	0.304	0.344	0.562	0.348	No	No
		GKlb	286	0.903	0.927	0.364	0.385	0.470	0.452	No	No
		GKm	369	<0.001	<0.001	0	0	0.540	0.504	Yes	No
		Rl	659	0	0	0	0	0.001	0	Yes	Yes
		dT	417	0.138	0.134	0.248	0.402	0.051	0	No	Yes
32–10	3.8 (3,368)	GKl	913	0.815	0.817	0.080	0.197	0.011	0.214	Yes	No
		GKlb	892	0.855	0.855	0.141	0.204	0.005	0.256	Yes	No
		GKm	1120	0	0	0	0	0.032	0.006	Yes	Yes
		Rl	2046	0	0	0	0	0	0	Yes	Yes
		dT	1615	0.999	1.000	0.463	0.466	0.439	0	No	Yes
	4.0 (2,169)	GKl	606	0.419	0.421	0.347	0.529	0.138	0.247	No	No
		GKlb	592	0.758	0.768	0.442	0.500	0.137	0.251	No	No
		GKm	739	0	0	0	0	0.252	0.023	Yes	Yes
		Rl	1333	0	0	0	0	0	0	Yes	Yes
		dT	1049	0.995	0.999	0.463	0.465	0.340	0	No	Yes

Discussion: Seismology

- Regional declustered catalogs generally don't look Poisson in time.
- Window-declustered catalogs can't be Poisson in space-time.
- Window-declustered catalogs generally don't seem to have exchangeable times, necessary condition for Poisson.
- No clear definition of foreshock, main shock, aftershock.
- All big shocks can cause damage and death. Physics doesn't distinguish main shocks from others. So why decluster?

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