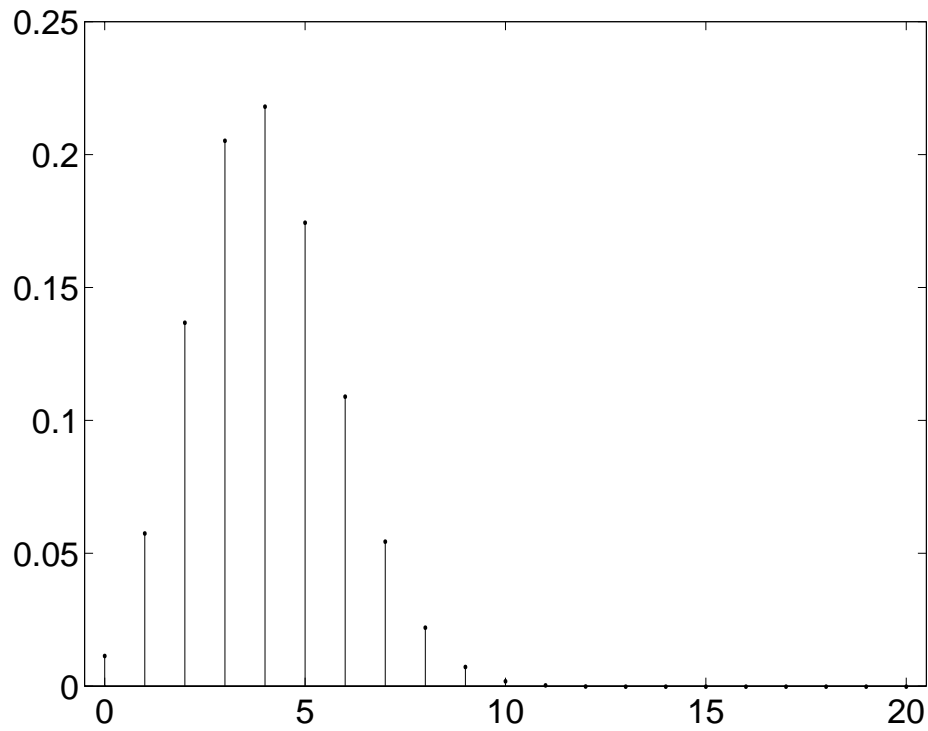
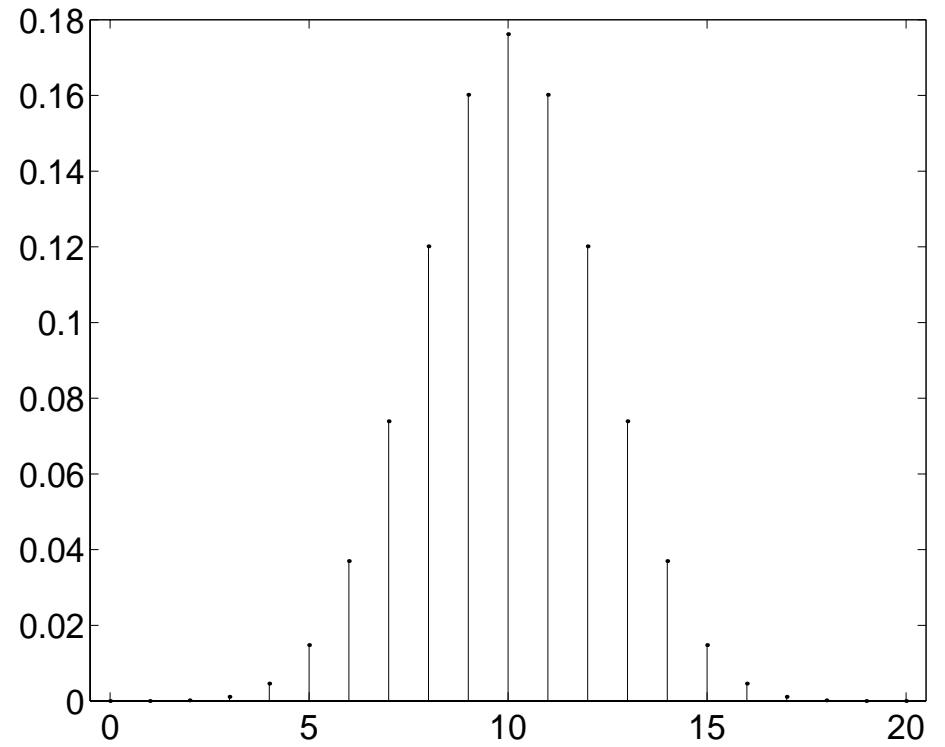


# The Law of Averages The Expected Value & The Standard Error

# Binomial Probabilities

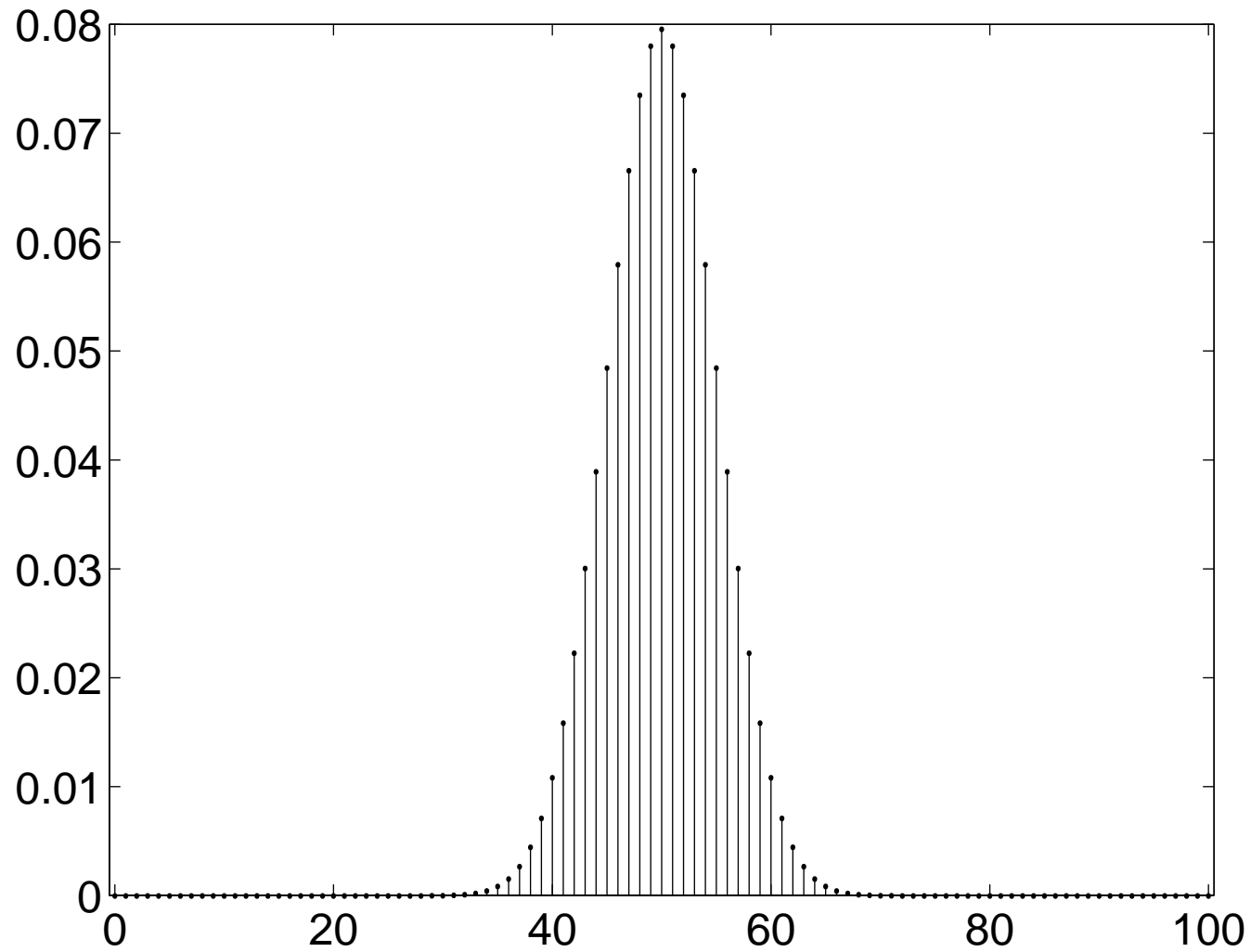


20 tries,  $p = .2$

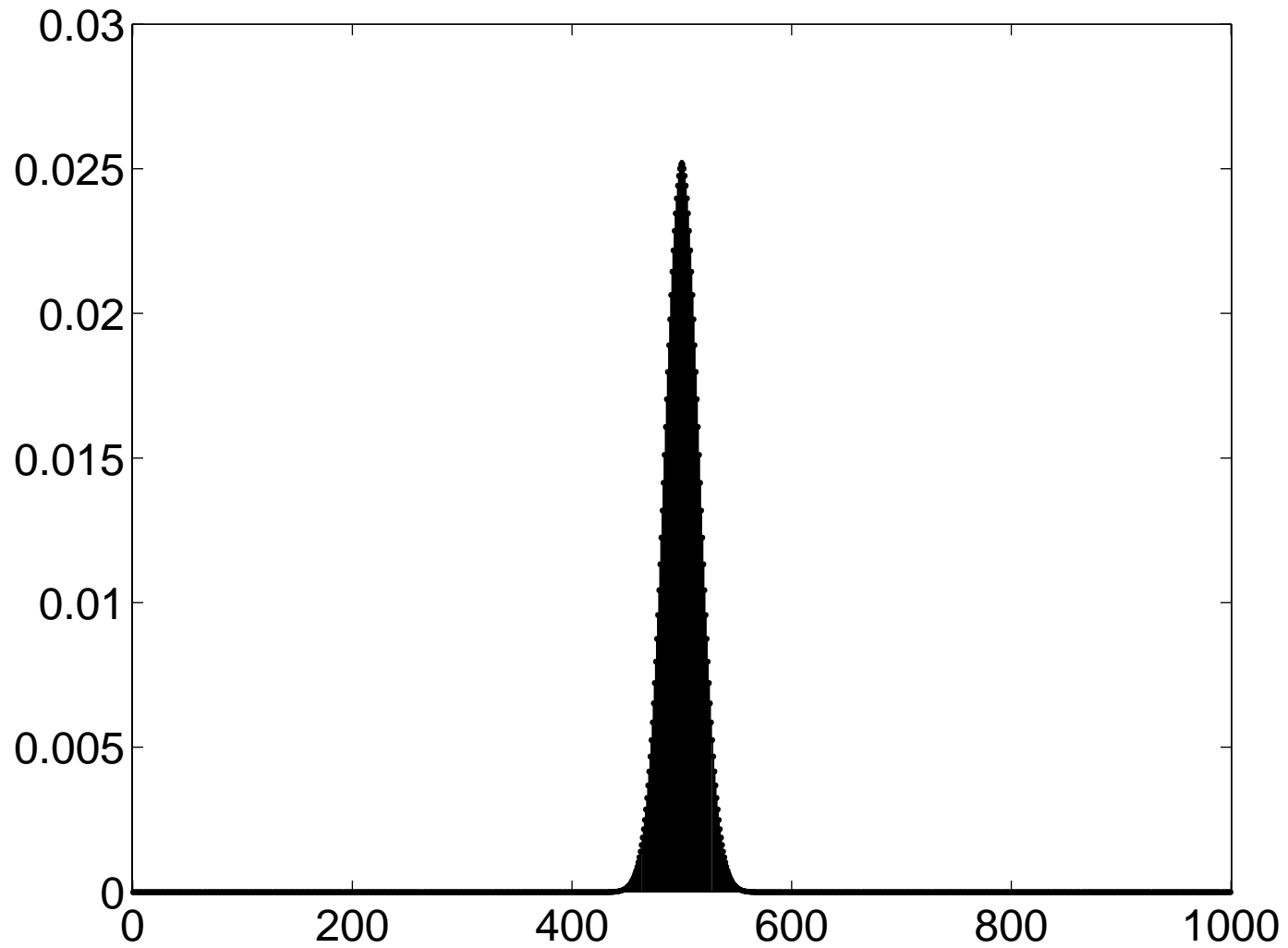


20 tries,  $p = .5$

# 100 Tries, $p = .5$



1 000 Tries,  $p = .5$



# The Law of Averages

As the number of tosses increases, the fraction of heads tends to a constant.

Number of heads = half the number of tosses  
+ chance error

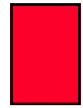
The chance error does not tend to zero, but the chance error divided by the number of tosses does.

# Box Models for Chance Processes: Roulette

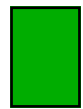
Wheel has 18 red, 18 black, and 2 green slots.  
You can bet on black or red, a specific number,  
or several other choices. When betting on black  
or red you either win or lose \$1.



18 black tickets



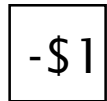
18 red tickets



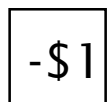
2 green tickets



18 black tickets



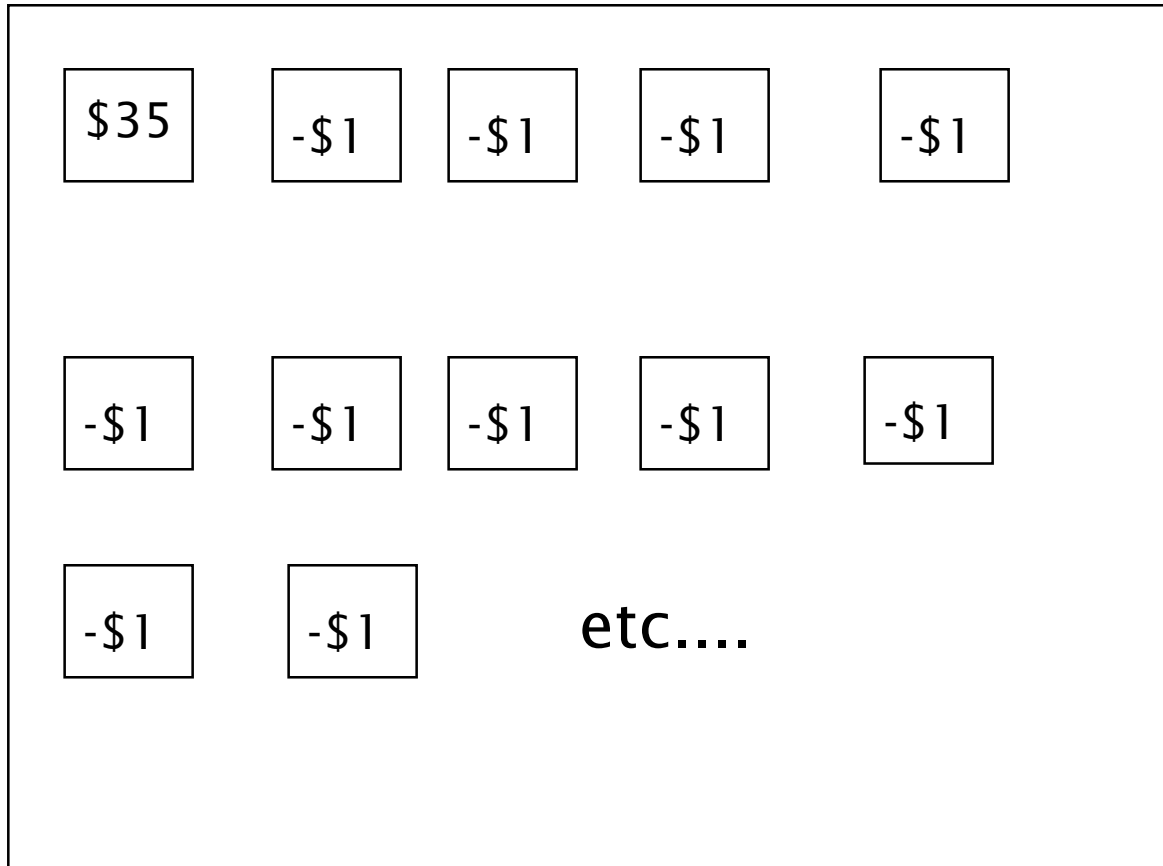
18 red tickets



2 green tickets

Your winning is the value of the ticket you draw from the box. If you play several times, your total winning is the sum of the values.

# Bet on a Number: 00,0,1,2,...,36



# California Lottery: Win and Spin

135,000,000 tickets  
Buy a ticket for \$1

Prize	Number of Tickets
\$1	10,800,000
\$2	8,100,000
\$5	3,240,000
\$10	540,000
\$50	54,000
\$100	27,000
\$500	6,073
\$1000	1,350
\$10,000	150

# The Expected Value

\$1 18 black tickets

-\$1 18 red tickets

-\$1 2 green tickets

In 100 draws you expect

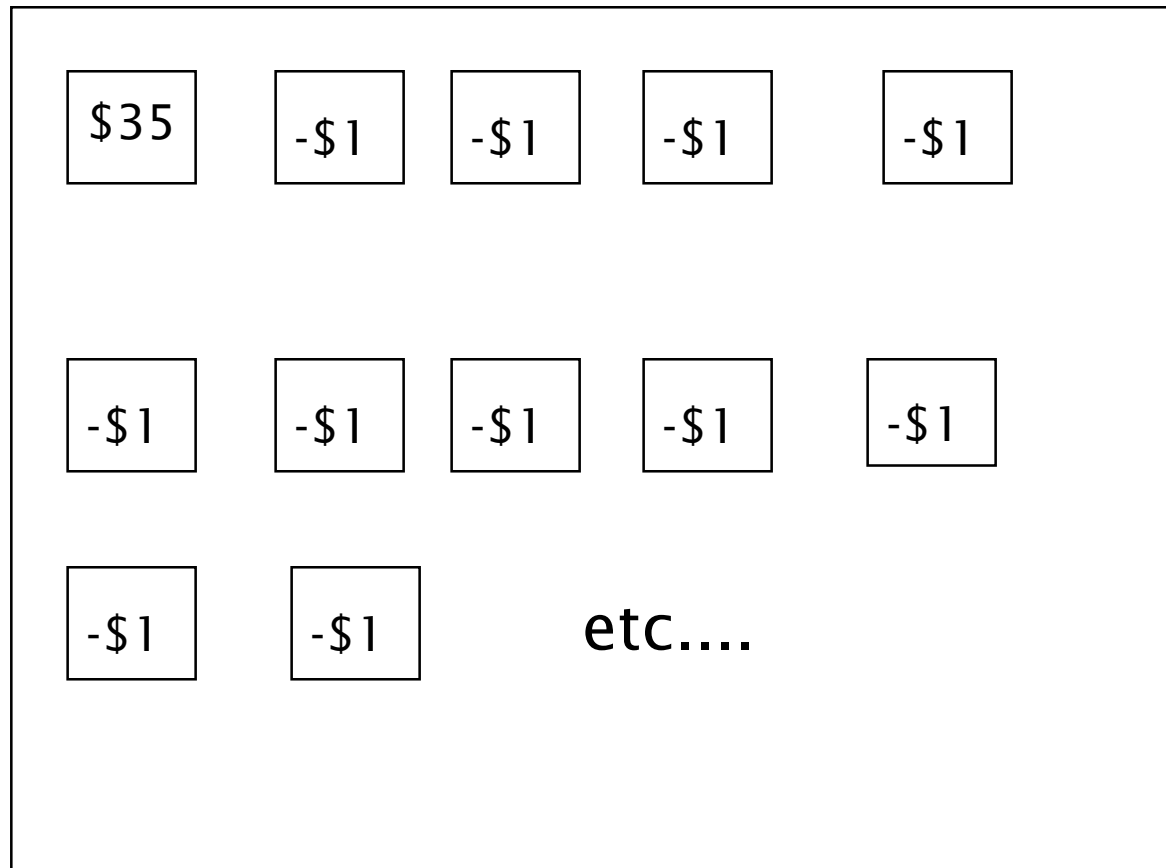
- 100 x (18/38) black (+\$1)
- 100 x (18/38) red (-\$1)
- 100 x (2/38) green (-\$1)

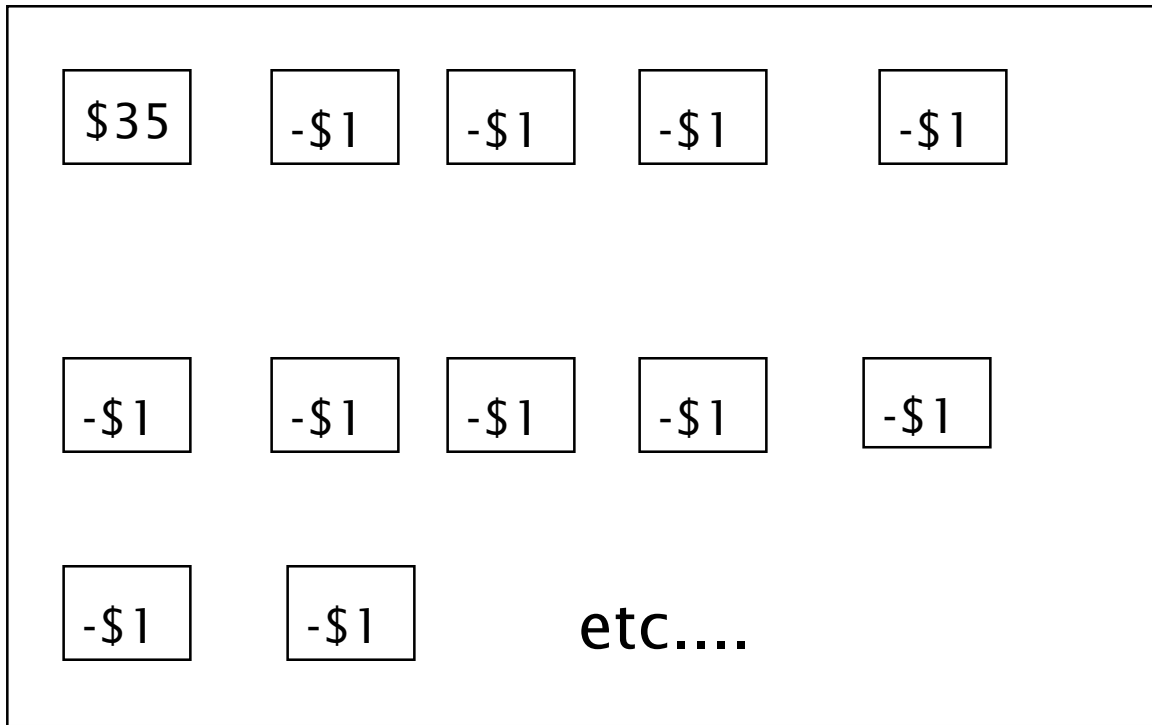
$$= 100 \times \frac{(\$1 \times 18) + (-\$1 \times 18) + (-\$1 \times 2)}{38} = -\$5.30$$

= 100 x average value of tickets in the box

The expected value for a sum of draws from a box equals the number of draws times the box average.

# Bet on a Number: 00,0,1,2,...,36





$$\text{Box average} = \frac{-\$1 \times 37 + \$35}{38} = \text{-.053\$}$$

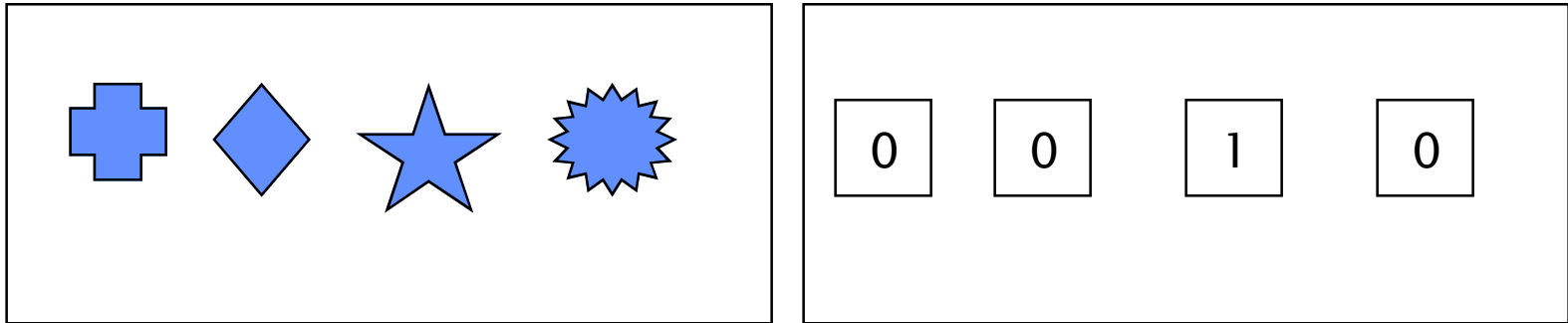
In 100 draws you expect to lose \$5.30

Prize	Number of Tickets
\$1	10,800,000
\$2	8,100,000
\$5	3,240,000
\$10	540,000
\$50	54,000
\$100	27,000
\$500	6,073
\$1000	1,350
\$10,000	150

Your winning equals the prize value minus \$1. So the values of the tickets equal the prize values minus \$1. The rest of the 135,000,000 tickets have value -\$1.

$$\text{Box Average} = -\$0.56$$

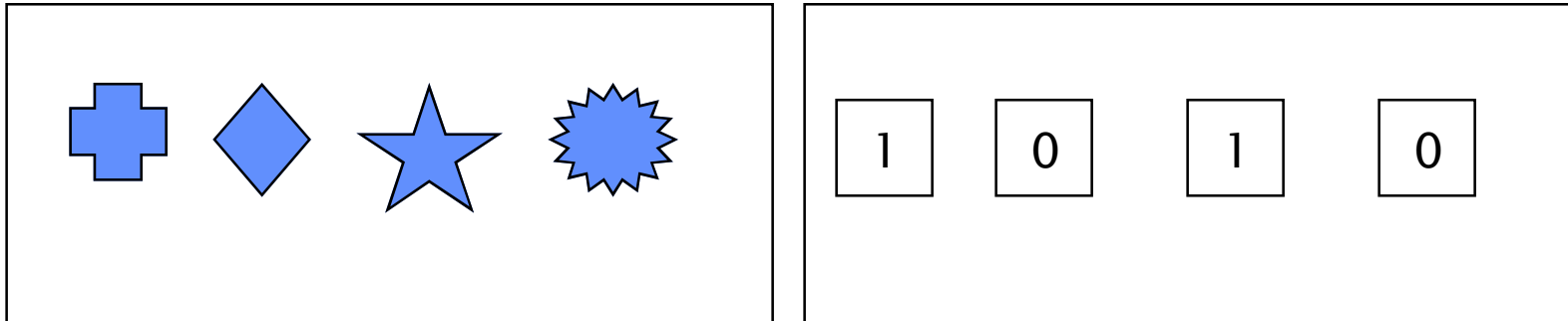
# Box Models for Counts: The Number of Times Do You Draw a



Box Average =  $1/4$

# Box Models for Counts: The Number of Times Do You Draw a

★ or a +



Box Average =  $2/4$

Sum = Expected Value + Chance Error

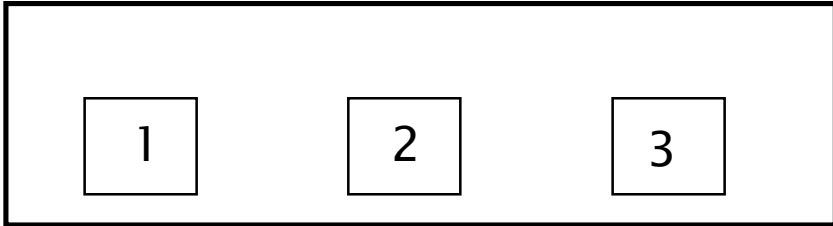
The **Standard Error (SE)** is a measure of how big the chance error is likely to be.

The **Square Root Law**: the standard error of the sum of draws is

$$\sqrt{\text{number of draws}} \times (\text{SD of the box})$$

# Conditions

- The draws are all from the same box.
- The draws are independent (with replacement).



Average of Box = 2

SD of Box = .8

4 Draws (SE = 1.6)

Sums	9	10	6	7	10	10	8	8	10	9
Chance Errors	1	2	-2	-1	2	2	0	0	2	1

16 Draws (SE = 3.2)

Sums	32	30	36	34	25	29	33	26	34	33
Chance Errors	0	-2	4	2	-7	-3	1	-6	2	1

64 Draws (SE = 6.4)

Sums	139	134	126	128	126	125	122	136	130	119
Chance Errors	11	6	-2	0	-2	-3	-6	8	2	-9

# Roulette: Betting on Black

Box Average =  $-.0526$

SD of Box =  $.99$

Draw 100 times

Expected value of sum =  $-\$5.26$       SE of sum of =  $9.9$

Chance error = sum -  $\$5.26$

**Sums:** 11    6    -2    0    -2    -3    -6    8    2    -9

**Errors** -8.74   -6.74   9.26   -18.74   -2.74   9.26   -6.74   13.26   17.26   -2.74

# Roulette: Betting on a Number

Box contains 1 ticket worth \$35 and 37 tickets worth -\$1

Box Average =  $-.0526$

Box SD = \$5.75

Draw 100 times

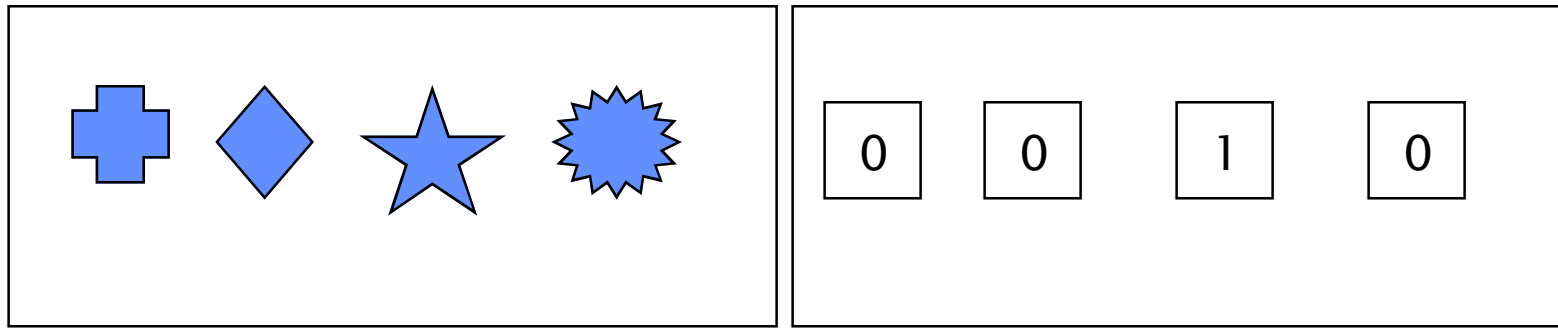
Expected value of sum = ?

SE of sum of = ?

Chance error = sum - expected value

<b>Sums:</b>	-64	8	-28	-100	-28
	-64	-64	152	44	-28
<b>Errors:</b>	-58.74	13.26	-22.74	-94.74	-22.74
	-58.74	-58.74	157.26	49.26	-22.74

# Box Models for Counts: The Number of Times You Draw a

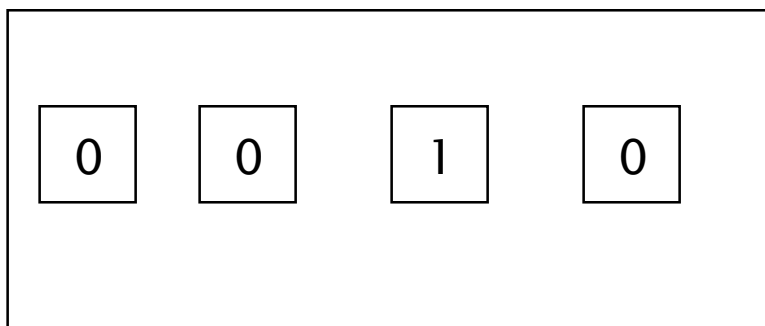


Box Average =  $1/4$

Special Formula for SD of a 0-1 box:

$$\sqrt{(\text{fraction of 1's}) \times (\text{fraction of 0's})}$$

For this box,  $SD = \sqrt{(1/4) \times (3/4)} = .87$



Box Average =  $1/4$

Box SD = .87

Suppose you draw 100 times

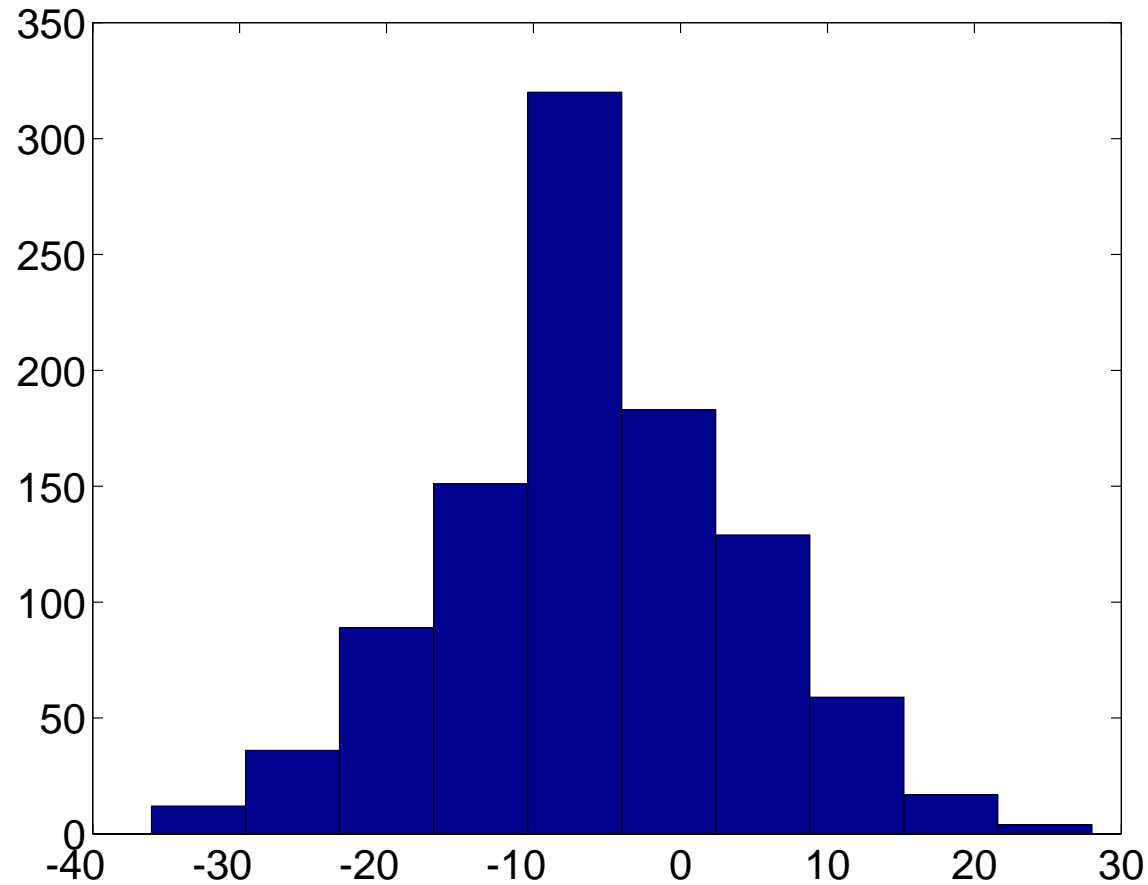
How many ★ would you expect?

What is the SE of the number of ★ ?

Would you be surprised by 30? By 70?

# Roulette: Betting on Black

## 1 000 Simulations of Sums of 100 Draws



Expected Value of Sum =  $-\$5.30$

SE of Sum =  $\$9.90$

# Roulette: Using the Normal Curve

Bet on Black

Box contains 18 tickets = \$1 and 20 tickets = -\$1

Box Average =  $-.0526$

SD of Box =  $.99$

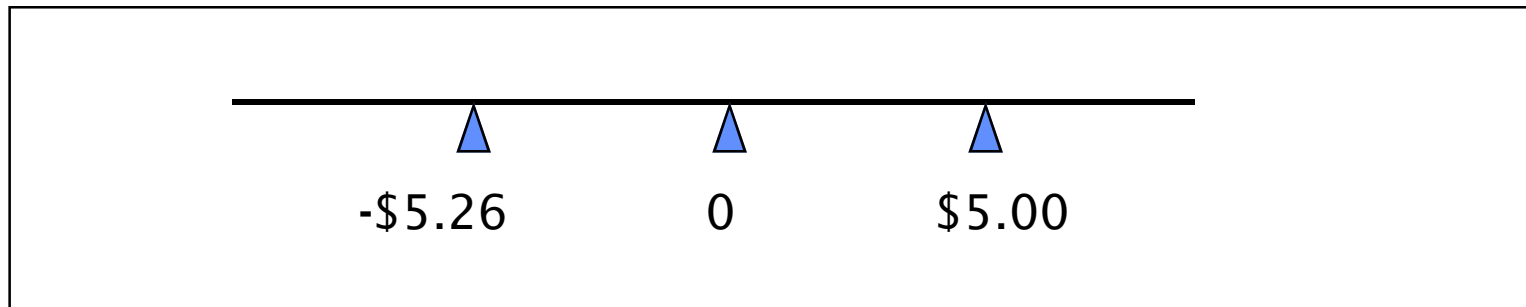
Draw 100 times

Expected value of sum =  $-\$5.26$

SE of sum of =  $9.9$

What's the chance of winning \$5 or more? Use normal curve to get an approximate answer.

Convert to Standard Units:



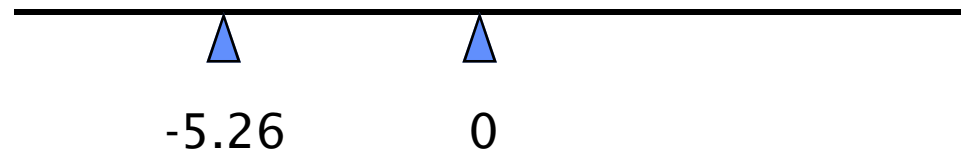
Expected Value =  $-\$5.26$ ; SE =  $\$9.90$

$10.26/9.9 = 1.0$  standard units.

From the normal table, the chances are about 15%

# What's the Chance You Come Out Ahead?

Expected Value =  $-\$5.26$ ; SE =  $\$9.90$



$5.26/9.9 = .53$  standard units.

From the table of the normal curve, the chance is about 29%.

# Is it better to play 100 times for \$1 or 1 time for \$100?

If you play 100 times, your expected winnings are -\$5.26.

If you play once:

- you lose \$100 with probability  $20/38$
- you gain \$100 with probability  $18/38$
- Expected winnings =  $-100 \times (20/38)$   
 $+ 100 \times (18/38)$   
 $= -\$5.26$

However, if you play 100 times for \$1 each time, the chance you come out ahead is about 29%.

If you play once for \$100, the chance you come out ahead is  $18/38 = 47\%$