The Binomial Formula

$$\frac{n!}{k!\,(n-k)!}\; p^{k}(1-p)^{n-k}$$

Suppose that genders of successive children are independent. Four children.

Suppose that chance of boy = 1/2. What is the chance of BBGG? BGBG? BGGG?

Suppose that chance of boy = 1/3. What is the chance of BBGG? BGBG? BGGG?

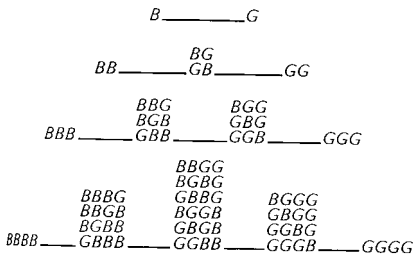
How many ways can you have three boys?

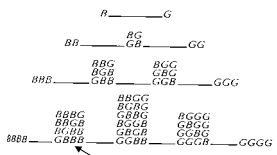
BBBG BBGB BGBB GBBB

What's the chance of having three boys?

(# ways) x probability of each way

Possible families: listed in birth order

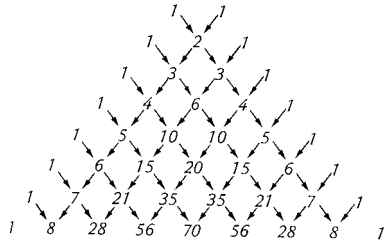




If there are 4 children, there are 4 ways to have 3 boys.

If chance of a boy = 1/3, chance of 3 boys in 4 children is

Pascal's Triangle



Blaise Pascal
(1623-1662)

A brilliant mathematician and physicist interested in probability and in the infinite. Also a mystic.

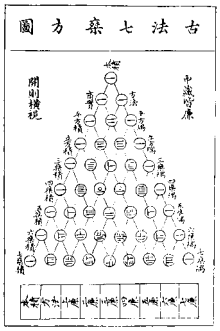
When his contemporaries resisted using infinitely small and infinitely large objects in math, he said, "The heart intervenes to make things clear."

Pascal's wager: "Let us weigh the gain and the loss in wagering that God is. If you gain, you gain all. If you lose, you lose nothing. Wager then, without hestitation that he is."

More quotations from Pascal

- Had Cleopatra's nose been shorter, the whole history of the world would have been different.
- Man is only a reed, the weakest thing in nature, but a thinking reed.
- The heart has its reasons, which reason knows nothing of.
- I have made this letter longer than usual, because I lack the time to make it short.

Chu Shih-Chieh's Triangle



From Ssu Yuan Yu Chien
by Chu Shih-Chieh,
written in 1303 in China

The Formula

Two possible outcomes: call them "success" and "failure."

The number of ways of having 3 successes in 4 trials (SSSF SSFS SFSS FSSS):

$$\frac{4 \times \cancel{3 \times 2 \times 1}}{\cancel{(3 \times 2 \times 1)} \times 1} = \frac{4!}{3! \times 1!} = 4$$

You can cancel

$$\frac{(\#\text{ trials})!}{(\#\text{successes})! \times (\#\text{failures})!}$$

Example

A class has 100 students. How many ways are there to make up a section of 25 students?

Identify being in the section as a "success."

The chance that of getting k successes in n tries is

$$\frac{n!}{k!\,(n-k)!}\,p^{k}(1-p)^{n-k}$$

$\frac{n!}{k!\,(n-k)!}$: # ways

$p^{k}(1-p)^{n-k}$: chance of each way

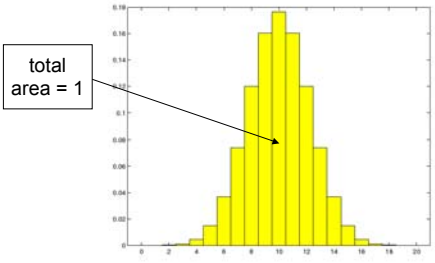
If the tries are *independent* and the chance of success on one try is p.

Example

Suppose the stock market goes up or goes down with probability 1/2 each day, independently* of whether it went up or down the previous days. What is the chance that it goes up four out of five successive days?

Binomial Probability Histograms

n=20 p = .5



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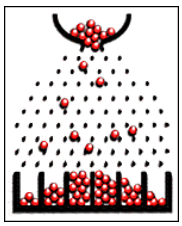
The Quincunx



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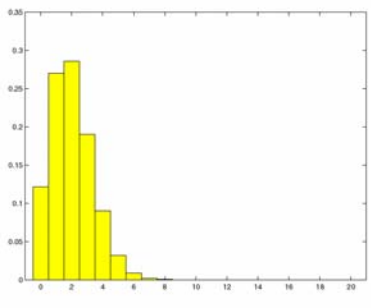
N=20 p = .1



More Binomial Pictures

Notation and Terminology

$$\frac{n!}{k!(n-k)!} = \binom{n}{k}$$ binomial coefficient

This is the number of ways k objects can be selected from n objects

A random number is often called a *random variable*.

A random variable generated by the binomial probability law, or "binomial distribution," is called a *binomial random variable*.[17]

Recap

If there are a *fixed* number of trials, with *independent* outcomes, each with the *same* probability of success, then the chance of a given number of successes in the sequence is given by the binomial probability formula.

Some examples where the binomial probability formula does not apply:

•Five dart players each throw one dart. The number of bull's eyes is recorded.

•The number of successful suicide attempts in a city in a month.

- The number of times your phone rings in an evening.
- The number of hearts you are dealt in a poker hand of seven cards.
- The number of students who successfully answer a problem on a midterm.
- The number of times you call your girlfriend (boyfriend) before he (she) finally picks up the phone.

Some Examples

A die is thrown 10 times

1. What is the chance of 8 6's?

2. At least 8 6's?

2. At most 8 6's?