A test statistic is calculated in the following way. First, we group all $m + n$ observations together and rank them in order of increasing size (we will assume for simplicity that there are no ties, although the argument holds even in the presence of ties). We next calculate the sum of the ranks of those observations that came from the control group. If this sum is too small or too large, we will reject the null hypothesis.

It is easiest to see how the procedure works by considering a very small example. Suppose that a treatment and a control are to be compared: Of four subjects, two are randomly assigned to the treatment and the other two to the control, and the following responses are observed (the ranks of the observations are shown in parentheses):

Treatment	Control
1 (1)	6 (4)
3 (2)	4 (3)

The sum of the ranks of the control group is $R = 7$, and the sum of the ranks of the treatment group is 3. Does this discrepancy provide convincing evidence of a systematic difference between treatment and control, or could it be just due to chance? To answer this question, we calculate the probability of such a discrepancy if the treatment had no effect at all, so that the difference was entirely due to the particular randomization—this is the null hypothesis. The key idea of the Mann-Whitney test is that we can explicitly calculate the distribution of R under the null hypothesis, since under this hypothesis every assignment of ranks to observations is equally likely and we can enumerate all $4! = 24$ such assignments. In particular, each of the $\binom{7}{2} = 6$ assignments of ranks to the control group shown in the following table is equally likely:

Ranks	R
$\{1, 2\}$	3
$\{1, 3\}$	4
$\{1, 4\}$	5
$\{2, 3\}$	5
$\{2, 4\}$	6
$\{3, 4\}$	7

From this table, we see that under the null hypothesis, the distribution of R (its null distribution) is:

r	3	4	5	6	7
$P(R = r)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$

In particular, $P(R = 7) = \frac{1}{6}$, so this discrepancy would occur one time out of six purely on the basis of chance.