Thus,

$$-2 \log \Lambda \approx 2n \sum_{i=1}^{m} [\hat{p}_i - p_i(\hat{\theta})] + n \sum_{i=1}^{m} \frac{[\hat{p}_i - p_i(\hat{\theta})]^2}{p_i(\hat{\theta})}$$

The first term on the right-hand side is equal to 0 since the probabilities sum to 1, and the second term on the right-hand side may be expressed as

$$\sum_{i=1}^{m} \frac{[x_i - np_i(\hat{\theta})]^2}{np_i(\hat{\theta})}$$

since x_i, the observed count, equals $n\hat{p}_i$ for $i = 1, \ldots, m$.

We have argued for the approximate equivalence of two test statistics. Pearson's test has been more commonly used than the likelihood ratio test, because it is somewhat easier to calculate without the use of a computer.

Let us consider some examples.

EXAMPLE **A** *Hardy-Weinberg Equilibrium*

Hardy-Weinberg equilibrium was first introduced in Example A in Section 8.5.1. We will now test whether this model fits the observed data. Recall that the Hardy-Weinberg equilibrium model says that the cell probabilities are $(1-\theta)^2$, $2\theta(1-\theta)$, and θ^2. Using the maximum likelihood estimate for θ, $\hat{\theta} = .4247$, and multiplying the resulting probabilities by the sample size $n = 1029$, we calculate expected counts, which are compared with observed counts in the following table:

	Blood Type		
	M	MN	N
Observed	342	500	187
Expected	340.6	502.8	185.6

The null hypothesis will be that the multinomial distribution is as specified by the Hardy-Weinberg equilibrium frequencies, with unknown parameter θ. The alternative hypothesis will be that the multinomial distribution does not have probabilities of that specified form. We first choose a value for α, the significance level for the test (recall that the significance level is the probability of falsely rejecting the hypothesis that the multinomial cell probabilities are as specified by genetic theory). In this application, there is no compelling reason to choose any particular value of α, so we will follow convention and let $\alpha = .05$. This means that our decision rule will falsely reject H_0 in only 5% of the cases.

We will use Pearson's chi-square test, and therefore X^2 as our test statistic. The null distribution of X^2 is approximately chi-square with 1 degree of freedom. (There are two independent cells, and one parameter has been estimated from the data.) Since, from Table 3 in Appendix B, the point defining the upper 5% of the chi-square distribution with 1 degree of freedom is 3.84, the test rejects if $X^2 > 3.84$. We next