Stat 155 Game theory, Yuval Peres **Fall 2004**

Homework 4

Question 1: The game of *hex* is played on an array of hexagons which has the shape of a rhombus which a 60 degree angle between successive sides, with equal numbers of hexagons lying along each of the four sides of the rhombus. We label these sides 1,2,3 and 4 in anticlockwise order. Player I puts down red hexagons, player II black. Player I aims to aim a red crossing from side 1 to side 3, player II, a black one from side 2 to side 4. This exercise is to prove that player I has a winning strategy by using the idea of strategy stealing that was used to solve the game of chomp. The first step is to show that from any position, one of the players has a winning strategy. In the second step, assume that player II has a winning strategy, and derive a contradiction.

Question 2*: the bomber and the submarine. In the game G_n, there are $n+1$ times, labelled 0,1, up to n. At one of these times, a bomber chooses to drop a bomb at an integer point. At any given time, a submarine, located at an integer point, moves one space to the left, or one to the right. A bomb takes exactly two time steps to reach its target, which it always hits accurately. It destroys the submarine if it hits that site at which the submarine is currently located, and does not otherwise. The last time at which the bomber can drop a bomb is n, so that the game may not be resolved until time $n+2$. Set up the payoff matrices and find the value of the games G_0, G_1 and G_2.

Question 3: two cities. There are two roads that leave city A and head towards city B. One goes there directly. The other branches into two new roads, each of which arrives in city B. A traveller and a troll each choose a path from city A to city B. The traveller will pay the troll a toll equal to the number of common roads that they traverse. Set up the payoff matrix in this case, find the value of the game, and find some optimal mixed strategies.

*This question has double weight.