**Stat 155 Game theory, Yuval Peres** **Fall 2004**

# Homework 3

**Question 1:** Recall the following recursive definition of a sequence of pairs of natural numbers: $(a_0, b_0) = (0, 0), (a_1, b_1) = (1, 2)$, and, for each $k \geq 1$,

$$a_k = mex\{a_1, \ldots, a_{k-1}, b_0, \ldots, b_{k-1}\}$$

and $b_k = a_k + k$. Prove carefully that the $(a_k, b_l)$ are up to symmetry the set of $P$-positions in the game of Wythoff's nim.

**Question 2*:** It was claimed in lectures that $a_k = \alpha_k(\theta)$ and $b_k = \beta_k(\theta)$, for some $\theta \in (0, 1)$. Here $\alpha_k(\theta) = \lfloor k/\theta \rfloor$ and $\beta_k(\theta) = \lfloor k/(1-\theta) \rfloor$. Find the value of $\theta$ by using the fact that $b_k = a_k + k$, and then dividing by $k$.

**Question 3: Chomp**. A chocolate bar made up a rectangular array of squares has poison in its southwest square. In a legal move, a player identifies a vertex and chomps off the chocolate lying in the northeast quadrant of that vertex, in such a way that some squares of chocolate are removed. The aim of the game is to leave the opponent with only the poisoned square remaining. Show that the first player wins from a rectangular starting position.

**Question 4:**. From Ferguson, Chapter II, section 1.5, questions 1 and 2.

*This question is optional. Full credit may be obtained without answering it.