Practice Midterm: Statistics 155 - due Oct 18, 2004

Instructor: Yuval Peres   Duration: 75 minutes.

Instructions: Please write your name on **every** page. This examination contains three problems with weight 34 points each. Write each answer **very clearly** below the corresponding question. (Use back of page if needed). No books, notebooks or other written materials are allowed.

Good Luck!

1. Define precisely the Sprague-Grundy function g for a progressively finite impartial game. Consider the game which is played with piles of chips like nim, but with the additional move allowed of breaking one pile of size $k > 0$ into two nonempty piles of sizes $i > 0$ and $k - i > 0$. Show that the Sprague-Grundy function g for this game, when evaluated at positions with a single pile, satisfies $g(3) = 4$. Find $g(1000)$, that is, g evaluated at a position with a single pile of size 1000.

   Given a position consisting of piles of sizes 13, 24 and 17, how would you play?

2. Find the value of the zero-sum game given by the following payoff matrix, and determine optimal strategies for both players:

$$\begin{pmatrix} 8 & 0 & 6 & 7 \\ 2 & 6 & 3 & 1 \end{pmatrix}$$

3. A zebra has four possible locations to cross the Zembezi river, call them a, b, c, d, arranged from north to south. A crocodile can wait (undetected) at one of these locations. If the Zebra and the Crocodile choose the same location, the payoff to the crocodile (that is, the chance it will catch the zebra) is 1. The payoff to the crocodile is 1/2 if they choose adjacent locations, and 0 in the remaining cases, when the locations chosen are distinct yet non-adjacent.

   (a) Write the matrix for this zero-sum game in normal form.

   (b) Can you reduce this game to a 2×2 game?

   (c) Find the value of the game (to the crocodile) and optimal strategies for both.